International Trade in an Uncertain World*

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Abstract

We develop a tractable quantitative model of international trade in which agents make bilateral investments in resilience under general equilibrium uncertainty. Under both complete and incomplete financial markets, we show that these bilateral investments solve a portfolio problem of choosing trade partners. Countries' risk profiles become determinants of trade flows, income and welfare, whose first moments are affected by the second moments of productivity and trade costs. Changes in global economic uncertainty have heterogeneous effects across countries, depending on how they affect real hedging opportunities. The opening of trade can raise or reduce income volatility, but is revealed-preferred to autarky.

Keywords: uncertainty, resilience, trade, welfare

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1 Introduction

Recent years have seen growing concerns about uncertainty and resilience. The aftermath of the COVID-19 pandemic highlighted the potential for large-scale transport disruptions. The resurgence of economic nationalism has heightened trade policy uncertainty. Increased geopolitical tensions have raised national security concerns about access to critical goods and minerals. Each of these sources of uncertainty involves aggregate shocks that have general equilibrium effects. We develop a tractable quantitative model of international trade in which agents make *ex ante* bilateral investments in resilience under such general equilibrium uncertainty. We consider settings with trade in financial assets (complete markets) and without trade in financial assets (incomplete markets). In both cases, we show that these bilateral investments solve a portfolio problem in general equilibrium. Countries' risk profiles become determinants of patterns of trade, income and welfare. As a result, the second moments of productivity and trade costs affect the first moments of trade, income and welfare. Changes in global economic uncertainty have heterogeneous effects across countries, depending on how they affect the ability to engage in real hedging through trade.

A key challenge in introducing general equilibrium uncertainty into quantitative trade models is that the general equilibrium conditions of these models are high-dimensional and non-linear, which makes it difficult to obtain analytical results for the mapping from the distribution of exogenous shocks to the distribution of endogenous variables. For this reason, most previous research on quantitative trade models assumes that agents have perfect knowledge of the realizations of all general equilibrium variables when making import and export decisions. However, in the presence of general equilibrium uncertainty, agents make decisions based on the distribution of the endogenous variables rather than their realizations.

Modeling general equilibrium uncertainty matters because much of the debate about resilience involves thinking about aggregate shocks that are likely to have general equilibrium effects. For example, shocks to productivity or the institutional environment in the United States would have general equilibrium implications for wages and prices around the world. Similarly, the main source of concern about the dependence of the United States on Taiwan for supplies of advanced semiconductors is about the aggregate costs of losing access to this source of supply in the face of growing geopolitical tensions between China and the United States.

Our first main contribution is to develop a tractable approach to incorporating general equilibrium uncertainty into the class of quantitative trade models with a constant trade elasticity. Since models with general equilibrium uncertainty can be intractable, researchers often either abstract from this uncertainty or consider stylized settings with a small number of countries. In contrast, we provide an analytical characterization of the properties of the distribution of our model's endogenous variables, allowing for many asymmetric countries, an arbitrary network of bilateral trade costs, and a general distribution of fundamental shocks to productivities and trade costs.

Our second main contribution is to allow risk-averse agents to make endogenous *ex ante* investments in import and export capacity that determine countries' resilience in the face of this general equilibrium uncertainty. Import and export capacity capture bilateral investments in business services, logistics, transportation and customer capital. Consumer love of variety implies that the representative agent has an incentive to disperse bilateral investments across trade partners. But the composition of this portfolio of bilateral investments is shaped by a risk-return trade-off. On the one hand, concentrating bilateral investments more towards the partner country with the best average characteristics raises expected real income. On the other hand, dispersing bilateral investments across a wider range of partner countries enhances the ability to import from partners that have worse average characteristics but that experience favorable realizations of these characteristics.

In the special case of no uncertainty, our model is isomorphic to a deterministic constant elasticity quantitative trade model. More generally, with uncertainty, bilateral investments and sourcing decisions depend not only on the expected values of the model's endogenous variables, but also on their variances and covariances. Importer n invests more in import capacity with exporter i if its goods are cheap (i) in expectation; (ii) when importer n's marginal utility is high; (iii) when importer n has high purchasing power; and (iv) when other goods in importer n's consumption bundle are expensive. Similarly, exporter i invests more in export capacity with importer n if it has higher bilateral expenditure (i) in expectation; (ii) when exporter i's marginal utility is high; (iii) when exporter i's cost of living is low (i.e., when revenue is valuable in real terms); and (iv) when exporter i's cost of production is low. Financial market structure affects the values of these variances and covariances, but they remain important in determining bilateral investments even under complete markets. Intuitively, complete markets ensure that the ratios of countries' marginal utilities of income are equalized across states of the world for any network of bilateral investments. But agents are not indifferent over alternative networks of bilateral investments, and have an incentive to choose a network that takes into account the cross-country correlation of marginal costs and revenues.

In the general equilibrium of our model, wages in each country are determined by a system of income accounting conditions, which equate each country's income with expenditure on its goods. We use this system of general equilibrium conditions to characterize the variances and covariances of the model's endogenous variables that determine bilateral investments. We take a second-order Taylor-series expansion in this system of general equilibrium conditions around countries' mean log productivities and trade costs to obtain analytical expressions for the first and second moments of endogenous wages. We characterize the Jacobian and Hessian of wages with respect to productivity and trade cost shocks in terms of expenditure shares and the trade elasticity. Under incomplete markets, we show that these expressions take the same form as in the deterministic Armington model, such that our results apply throughout the class of deterministic quantitative trade models with a constant trade elasticity. Under complete markets, the Jacobian and Hessian must be adjusted to take account of the state-contingent transfers. The general equilibrium of our model is a fixed point at which bilateral investments in import and export capacity based on the first and second moments of the endogenous variables generate distributions of these endogenous variables that are consistent with these first and second moments.

We assume competitive markets and that firms in each country are owned by the representative agent in that country. Under these assumptions, the competitive market allocation coincides with the allocation that would be chosen by a national planner who maximized welfare taking the *ex ante* distribution and *ex post* realization of international prices (the terms of trade) as given. Intuitively, since firms are owned by the representative agent, profits in each state of the world are weighted by the representative agent's stochastic discount factor, which ensures that allocations in the competitive market equilibrium are aligned with those that would be chosen by a national planner. The open-economy allocation features positive investments in import and export capacity with foreign trade partners, which implies that the national planner weakly prefers positive foreign investments to only making domestic investments. Therefore, by revealed preference, welfare is weakly higher in the open economy than in the closed economy.

Opening the closed economy to trade affects certainty-equivalent welfare through both the mean (first moment) and volatility (second moment) of real income. A modified version of the Arkolakis et al. 2012 (henceforth ACR) formula holds for mean real income, modified to take account of endogenous changes in domestic import and export capacity between the closed and open economies. The volatility of real income can either rise or fall following the opening of trade, depending on the covariance structure of domestic and foreign shocks. Nevertheless, the opening of trade weakly raises welfare by the revealed preference argument above.

We implement our quantitative analysis using data on 106 countries from 1993-2017. For expositional simplicity, we treat country productivities as stochastic and bilateral trade costs as deterministic in our quantitative analysis, but all of our analytical results hold under both trade cost and productivity uncertainty. Given data on international trade and production, we invert the general equilibrium conditions of the model to recover country productivities in each year and the matrix of bilateral trade costs for our benchmark year of 2017. We use these time-varying productivities to compute mean country productivity and the variance-covariance matrix of country productivities. We find that a global factor (first principal component) accounts for around 43 percent of the variance of country productivities. In our model, labor is the sole factor of production, which implies that country productivities also capture capital stocks and labor wedges. Therefore, this finding of a strong global factor is consistent with existing evidence in the international co-movement literature. In general, countries that are geographically closer have higher bilateral productivity covariances, such that uncertainty in productivity is correlated with other determinants of trade.

We use our quantitative model to evaluate the impact of changes in the structure of global economic uncertainty. Eliminating the correlation in country productivities (by setting the off-diagonal terms in the variance-covariance matrix to zero and holding variances and mean productivity constant) tends to raise welfare in countries that load positively on the global factor in the initial equilibrium. These countries experience an expansion in their ability to diversify risk through international trade, reallocate their investments away from the domestic market to foreign markets, and reduce their domestic expenditure shares. In contrast, removing the correlation in country productivities tends to reduce welfare in countries that load negatively on the global factor. These countries experience a reduction in their ability to diversify risk through international trade, reallocate their investments away from foreign markets to the domestic market, and increase their domestic expenditure shares.

A proportional increase in global uncertainty (an increase in the absolute magnitude of all variances and covariances holding constant mean productivity) reduces welfare in all countries, because the representative agent in each country is risk averse. The resulting decline in welfare tends to be larger in countries with more volatile productivity in the initial equilibrium, because a proportional increase in global uncertainty leads to a larger increase in the variance of productivity in these countries. When sourcing goods in the initial equilibrium, countries trade off sourcing from low-cost suppliers in expectation (often from the domestic market where no trade costs are incurred) against diversifying risk by sourcing goods from abroad (where trade costs are incurred but foreign productivity is imperfectly correlated with domestic productivity). As global uncertainty increases, this increases the incentive for risk diversification, which leads to a reallocation of investments away from the domestic market to foreign markets, and a reduction in domestic expenditure shares.

Changes in uncertainty in individual countries, such as the United States, propagate through the trade network. We consider an increase in the variance of U.S. productivity, holding constant mean U.S. productivity and the cross-country correlation structure. We find that countries with a positive covariance with the U.S. in the initial equilibrium tend to experience a decline in certainty-equivalent welfare. In these countries, the increase in the dispersion of U.S. productivity makes trade with the U.S. less attractive for risk diversification. In contrast, countries with a negative covariance with the U.S. in the initial equilibrium experience a rise in certainty-equivalent welfare. In these countries, the increase in the dispersion of U.S. productivity makes trade with the U.S. more attractive for risk diversification. The resulting changes in demand for U.S. goods, and reallocations of bilateral investment, lead to endogenous changes in the terms of trade between countries.

We find a similar pattern of analytical and quantitative results under both complete and incomplete financial markets. Although complete markets ensure that the ratios of countries' marginal utilities of income are equalized across states of the world, the representative agent in each country has an incentive to target bilateral investments based on the cross-country correlation of costs and revenues. We show that the welfare gains from real hedging through bilateral investments are sizable relative to those from financial hedging through state-contingent claims. We find that real and financial hedging are complementary, with larger welfare gains from real hedging under complete markets than under incomplete markets. Since the bilateral co-movement in countries' productivities is correlated with the bilateral distance between countries, we show that the estimated distance coefficient in the gravity equation for international trade depends on the structure of global uncertainty and whether financial markets are complete or incomplete.

Our paper is related to several areas of existing research. First, we contribute to research on quantitative trade models with a constant trade elasticity, including Eaton and Kortum (2002), Arkolakis et al. (2012) and Kleinman et al. (2024). This research typically abstracts from general equilibrium uncertainty, because the system of general equilibrium conditions in these models is high-dimensional and non-linear. Caliendo et al. (2025) uses a Ricardian trade model to study the relationship between tariffs and trade deficits under uncertainty. Fan and Luo (2025) develops an extension of Ricardian trade models, in which the representative consumer sources varieties after observing the comparative advantage of a country for each variety, but before observing the realization of the country's aggregate productivity and bilateral trade costs. Ramondo and Ma (2026) analyzes how regional specialization and welfare are affected by uncertainty and economies of scale in the open economy. Whereas most of this research is concerned with first moment shocks, we develop a multi-country quantitative framework that can be used to evaluate second moment shocks, in which agents can make investments in resilience in the shadow of this uncertainty.

Second, we connect with research in macroeconomics, transportation economics and international trade on the propagation of shocks through networks, including Acemoglu et al. (2012), Baqaee and Farhi (2019), Liu (2019), Carvalho et al. (2021), Baqaee and Farhi (2024), Nikolakoudis (2024), Huo et al. (2025), Fuchs et al. (2026), and Brancaccio et al. (2026). Within this literature, Kopytov et al. (2024) examines supply chain uncertainty in a Cobb-Douglas production network, while Liu et al. (2024) develops a model of supplier capital, in which supply chain disruptions have heterogeneous effects across firms. In contrast, we develop a multi-country quantitative model of international trade, in which uncertainty has general equilibrium effects, and agents invest in resilience anticipating this uncertainty.

Third, an emerging strand of research within the networks literature has examined network fragility and resilience. Elliott et al. (2022) allows firms to multi-source inputs and strategically invest to strengthen relationships, trading off the cost of investment against the benefits of increased robustness. Grossman et al. (2024) analyzes optimal resilience in vertical supply chains, in which firms choose investments in protective capabilities and whether to form supply links, and undertake sequential bargaining over quantities and payments. Castro-Vincenzi et al. (2025a) models sequential production with supply chain risk.[1] Castro-Vincenzi et al. (2025b) provides theory and evidence on the role of climate risk in shaping firm supply chains in the face of climate risk. Relative to this research, we provide an analytical characterization of the impact of uncertainty on trade, income and welfare in quantitative trade models, allowing for general equilibrium changes in wages, investments in resilience, and either complete or incomplete financial markets.

Fourth, our paper is related to the wider literature on trade and uncertainty. Helpman and Razin (1978) and Helpman (1988) analyze theoretically the determinants of trade in securities, goods and factor services under uncertainty. Newbery and Stiglitz (1984) shows that the opening of trade can be welfare reducing under uncertainty if it induces greater volatility in prices in the open economy than in the closed economy. In contrast, Caselli et al. (2020) argues that trade can reduce income volatility if country-level shocks are more important than sector-level shocks.[2] Allen and Atkin (2022) examines the welfare effects of trade through first and second moments using a model of agricultural land specialization for Indian districts. Compared to this research, we examine uncertainty and endogenous investments in resilience in multi-country quantitative models of the global economy.

Fifth, our work connects with the international macroeconomics literature on international risk diversification. Cole and Obstfeld (1991) shows that trade in goods can contribute to international risk diversification. Obstfeld and Rogoff (2000) argues that the costs of trading goods play a central role in explaining the six major puzzles in international macroeconomics. Following Backus et al. (1992) a large literature analyzes the degree of international risk sharing.[3] Fitzgerald (2025) argues that the welfare gains from trade across goods, states and time are larger than implied by models of trade in goods alone. In contrast, we analyze the implications of general equilibrium uncertainty for trade patterns, income and welfare in multi-country quantitative trade models, incorporating investments in resilience, and allowing for either complete or incomplete financial markets.

Methodologically, the techniques that we use to characterize the effects of uncertainty on bilateral trade patterns are related to those used in the international portfolio literature (see, e.g., Heathcote and Perri 2013 and Coeurdacier and Gourinchas 2016) and in heterogeneous agent models in Auclert et al. (2024) and Bhandari et al. (2023). We use these techniques to analyze the role of risk in shaping comparative advantage and goods trade. By starting from the class of trade models with a constant trade elasticity, and making minimal departures, we develop an approach that remains tractable in a many-country setting, and lends itself to quantitative analysis.

The remainder of the paper is structured as follows. Section 2 introduces the economic environment that we use throughout the paper. Section 3 shows that the conventional deterministic

[1] Khanna et al. (2022) and Balboni et al. (2024) provide empirical evidence on resilience to supply chain shocks.

[2] Other research examines the impact of uncertainty in models of investment with sunk costs, including Dixit and Pindyck (1994), Bloom (2009), Handley and Limao (2017), and Alessandria et al. (2024).

[3] See for example Backus and Smith (1993), Colacito and Croce (2011), Geromichalos and Simonovska (2014), Baley et al. (2020), Kucheryavyy (2021), Aguiar et al. (2025), and Heiland (2025).

Armington model corresponds to a special case of this economic environment. Section 4 solves for bilateral investments in resilience and trade shares under uncertainty in terms of the first and second moments of the model's general equilibrium variables. Section 5 derives analytical expressions for these moments without trade in financial assets (incomplete markets). Section 6 derives the analogous expressions for these moments with trade in financial assets (complete markets). To illustrate our approach as clearly as possible, we focus in our baseline specification on a single sector. Section 7 shows that our approach generalizes to multiple sectors. Section 8 introduces our data. Section 9 reports our quantitative results. Section 10 summarizes our conclusions.

2 Economic Environment

We begin by summarizing the economic environment in our model, before solving for equilibrium under alternative assumptions about uncertainty (deterministic and stochastic model primitives) and trade in financial assets (incomplete and complete markets). For expositional simplicity, we model international trade following Armington (1969), in which goods are differentiated by country of origin. But our results hold throughout the class of constant elasticity quantitative trade models, including the Ricardian model of Eaton and Kortum (2002), the new trade model of Krugman (1980), and the heterogeneous firm model of Melitz (2003) and Chaney (2008) with a Pareto firm productivity distribution. Throughout the paper, we use bold math font to denote vectors or matrices. We report the derivations for each section of the paper in the Online Appendix.

2.1 Preferences and Endowments

We consider a world of many countries indexed by $n, i \in \{1, \ldots, N\}$. We index states of the world by ν, where a state corresponds to a set of realizations for country productivities $\{z_i(\nu)\}_{i=1}^{N}$ and bilateral trade costs $\{\tau_{ni}(\nu)\}_{n=1,i=1}^{N,N}$. We denote the probability distribution of each state by $g(\nu)$, where this probability distribution is degenerate in the absence of uncertainty. The representative agent in country n has labor endowment $\bar{\ell}_n$. She has constant relative risk aversion (CRRA) preferences over an aggregate consumption index ($\mathcal{C}_n(\nu)$):

$$\mathcal{U}_n(\nu) = \frac{\mathcal{C}_n(\nu)^{1-\gamma}}{1-\gamma}, \tag{1}$$

where $\gamma > 0$ is the coefficient of relative risk aversion. The aggregate consumption index in country n ($\mathcal{C}_n(\nu)$) is defined over the consumption of the good produced by each country i ($c_{ni}(\nu)$) and is assumed to take the constant elasticity of substitution (CES) form:

$$\mathcal{C}_n(\nu) = \left(\sum_{i=1}^{N} c_{ni}(\nu)^{\frac{\omega-1}{\omega}}\right)^{\frac{\omega}{\omega-1}}, \qquad \omega > 1, \tag{2}$$

where $\omega > 1$ is the elasticity of substitution between country varieties.

2.2 Trade Technology

Consuming a trade partner's good requires both imports and import capacity, where import capacity captures the importer's investments in business services, logistics, transportation and customer capital. Import capacity is chosen *ex ante* before observing the realizations for productivities and trade costs and hence does not depend on the state. If country n employs ι_{ni} workers in developing import capacity with exporter i, and sources $x_{ni}(\nu)$ units of that exporter's good net of variable trade costs, $c_{ni}(\nu)$ units can be consumed:

$$c_{ni}(\nu) = \iota_{ni}^{\alpha} x_{ni}(\nu)^{1-\alpha}, \qquad 0 \leq \alpha < 1. \tag{3}$$

Supplying a country's good to a trade partner requires both exports and export capacity, where export capacity captures the exporter's investments in business services, logistics, transportation and customer capital. Export capacity is also chosen *ex ante* before observing the realizations for productivities and trade costs and hence again does not depend on the state. If country i employs h_{ni} workers in developing export capacity with importer n, and ships $y_{ni}(\nu)$ units to that importer net of variable trade costs, $x_{ni}(\nu)$ units are supplied:

$$x_{ni}(\nu) = h_{ni}^{1-\widetilde{\delta}} y_{ni}(\nu)^{\widetilde{\delta}}, \qquad 0 < \widetilde{\delta} \leq 1. \tag{4}$$

Combining the importing technology (3) and the exporting technology (4), consumption $c_{ni}(\nu)$ by importer n of the good produced by exporter i is related to import capacity (ι_{ni}), export capacity (h_{ni}) and output net of variable trade costs ($y_{ni}(\nu)$) as follows:

$$c_{ni}(\nu) = \iota_{ni}^{\alpha} h_{ni}^{\beta} y_{ni}(\nu)^{\delta}, \qquad \alpha + \beta + \delta = 1, \tag{5}$$

where $\beta \equiv (1-\alpha)\left(1-\widetilde{\delta}\right)$; $\delta \equiv (1-\alpha)\widetilde{\delta}$; $0 < \beta < 1$; and $0 < \delta < 1$.

In this formulation (5), countries have access to a constant returns to scale trading technology in import capacity (ι_{ni}), export capacity (h_{ni}) and production net of variable trade costs ($y_{ni}(\nu)$). Import and export capacity correspond to non-traded services that are supplied to the production sector to enable its goods to be traded. We assume a Cobb-Douglas functional form for this trading technology (5) for tractability and three related reasons. First, this specification captures the idea from the networks literature that positive investments are required to activate relationships, because positive investments in import and export capacity are required for positive trade to occur. Second, this specification also captures the idea that importers and exporters can invest in deepening their relationship, because larger investments in import and export capacity reduce the bilateral costs of trading. Third, in the absence of uncertainty, this specification is isomorphic to the deterministic Armington model, as shown formally in Section 3 below.

2.3 Production Technology

Country i's good is produced using labor according to a constant returns to scale technology with productivity $z_i(\nu)$. There are iceberg bilateral trade costs, such that $\tau_{ni}(\nu) \geq 1$ units of a good must be shipped from country i in order for one unit to arrive in country n, with $\tau_{ni}(\nu) > 1$ for $n \neq i$ and $\tau_{nn}(\nu) = 1$. The output supplied by exporter i to importer n net of variable trade costs ($y_{ni}(\nu)$) is:

$$y_{ni}(\nu) = z_i(\nu)\,\ell_{ni}(\nu)/\tau_{ni}(\nu), \tag{6}$$

where $\ell_{ni}(\nu)$ is the quantity of labor in exporter i allocated to importer n.

In contrast to labor employed in import and export capacity, which is determined *ex ante*, labor employed in production ($\ell_{ni}(\nu)$) can be reallocated across trade partners *ex post*, after observing the realizations for productivities and bilateral trade costs.

2.4 Labor Market Clearing

Labor market clearing requires that the supply of labor in country i ($\bar{\ell}_i$) is equal to the sum across importers n of labor allocated to production ($\ell_{ni}(\nu)$) and export capacity (h_{ni}) plus the sum across exporters j of labor allocated to import capacity (ι_{ij}):

$$\bar{\ell}_i = \sum_{n=1}^{N}(\ell_{ni}(\nu) + h_{ni}) + \sum_{j=1}^{N} \iota_{ij}. \tag{7}$$

2.5 Market Equilibrium

All markets are assumed to be perfectly competitive, including the production, importing and exporting sectors. All firms in each country are owned by the representative agent in that country. We denote the wage in country n by $w_n(\nu)$ and aggregate expenditure in country n by $\mathcal{W}_n(\nu)$. Income accounting requires that income in each exporter i equals the sum across importers n of expenditure on the goods produced by that exporter:

$$w_i(\nu)\,\bar{\ell}_i = \sum_{n=1}^{N} s_{ni}(\nu)\,\mathcal{W}_n(\nu), \tag{8}$$

where $s_{ni}(\nu)$ is the share of importer n's expenditure on exporter i.

We allow for state-contingent transfers. Under incomplete markets, we exogenously set these state-contingent transfers to zero, such that each country is in financial autarky, with aggregate expenditure equal to aggregate income in each state ($\mathcal{W}_n(\nu) = w_n(\nu)\,\bar{\ell}_n$). Under complete markets, we allow agents to endogenously choose the quantity of state-contingent transfers, and allow their price to be endogenously determined by market clearing, such that aggregate expenditure in general differs from aggregate income in each state ($\mathcal{W}_n(\nu) \neq w_n(\nu)\,\bar{\ell}_n$).

3 Deterministic Armington Model

We begin by considering the special case of this environment that corresponds to the conventional deterministic Armington model. This special case corresponds to the assumption of no uncertainty over productivity or bilateral trade costs and no bilateral investments in import or export capacity ($\alpha = \beta = 0, \delta = 1$). These assumptions imply that the distribution of states ($g(\nu)$) is degenerate and hence we drop the implicit dependence of variables on state ν. With no state-contingent transfers, aggregate expenditure equals aggregate income ($\mathcal{W}_n = w_n \bar{\ell}_n$). Equilibrium import and export capacity are zero ($\iota_{ni} = h_{in} = 0$), such that $c_{ni} = x_{ni} = y_{ni}$.

Using cost minimization, zero profits, and expenditure minimization, we can write the income accounting relationship between a country's income and expenditure on its goods as:

$$w_i \bar{\ell}_i = \sum_{n=1}^{N} \frac{\left(w_i \tau_{ni} / z_i\right)^{-\theta^{DA}}}{\sum_{j=1}^{N} \left(w_j \tau_{nj} / z_j\right)^{-\theta^{DA}}} w_n \bar{\ell}_n. \tag{9}$$

In the deterministic Armington model (superscript DA), the trade elasticity is determined by the elasticity of substitution ($\theta^{DA} = \omega - 1$). General equilibrium reduces to solving for the N wages in each country (w_n) such that the system of N equations (9) holds. Given this solution for wages in each country, we can recover the equilibrium values of all other endogenous variables. Since country wages are gross substitutes in the system of equations (9), there exists a unique wage vector (up to a choice of numeraire) that solves this system of equations, as shown in Alvarez and Lucas (2007) and Allen et al. (2020). We choose world gross domestic product (GDP) as the numeraire: $\sum_{n=1}^{N} q_n = 1$, where we use $q_n = w_n \bar{\ell}_n$ to denote country GDP.

We next retain the assumption of no uncertainty but allow positive investments in import and export capacity ($\{\alpha, \beta, \delta\} \in (0, 1)$). Therefore, the distribution of states ($g(\nu)$) is again degenerate and hence we drop the implicit dependence on state ν. With no state-contingent transfers, aggregate expenditure equals aggregate income ($\mathcal{W}_n = w_n \bar{\ell}_n$). Equilibrium investments in import and export capacity are now non-zero ($\{\iota_{ni}, h_{ni}\} \in (0, \infty)$), such that $c_{ni} \neq x_{ni} \neq y_{ni}$.

With no uncertainty, the competitive market equilibrium of our model with import and export capacity is isomorphic to that in the deterministic Armington model. The income accounting relationship between a country's income and expenditure on its goods can be written in exactly the same form as in the conventional Armington model.

$$w_i \bar{\ell}_i = \sum_{n=1}^{N} \frac{\left(\bar{\tau}_{ni} w_i / \bar{z}_i\right)^{-\theta^{NU}}}{\sum_{j=1}^{N} \left(\bar{\tau}_{nj} w_j / \bar{z}_j\right)^{-\theta^{NU}}} w_n \bar{\ell}_n. \tag{10}$$

where we have defined $\theta^{NU} \equiv (1 - \alpha)(\omega - 1)$; $\bar{\tau}_{ni} \equiv \tau_{ni}^{\frac{1-\alpha-\beta}{1-\alpha}}$; and $\bar{z}_i \equiv z_i^{\frac{1-\alpha-\beta}{1-\alpha}}$. A key implication of this isomorphism is that the novel predictions of our model below do not arise from the introduction of bilateral investments in import and export capacity *per se*, but are rather driven by uncertainty and its interaction with these bilateral investments.

To establish this isomorphism, we have defined adjusted trade costs ($\overline{\tau}_{ni} \equiv \tau_{ni}^{\frac{1-\alpha-\beta}{1-\alpha}}$), which take into account the cost shares of import and export capacity (α, β) in overall consumption. The elasticity of trade flows with respect to unadjusted trade costs (τ_{ni}) also depends on these cost shares: $\theta^{LR} = \delta\left(\omega - 1\right) < \omega - 1$, where recall $\delta = 1 - \alpha - \beta$. This trade elasticity is lower than in the deterministic Armington model, because variable trade costs are incurred on traded goods, which are now only part of overall consumption. We interpret this trade elasticity as the *long-run* trade elasticity (θ^{LR}), because it captures the full response of trade with respect to variable trade costs (including adjustments in import and export capacity) after the resolution of all uncertainty.

4 Uncertainty and Resilience

We now introduce uncertainty, such that the probability distribution over states ($g\left(\nu\right)$) is non-degenerate, and productivities $\left\{z_i\left(\nu\right)\right\}_{i=1}^{N}$ and bilateral trade costs $\left\{\tau_{ni}\left(\nu\right)\right\}_{n=1,i=1}^{N,N}$ are random variables. We allow for bilateral investments in resilience (import and export capacity) that are undertaken before the realizations for productivities and bilateral trade costs are observed. These realizations for productivities and bilateral trade costs affect wages through the income accounting relationship between a country's income and expenditure on its goods. Therefore, the general equilibrium variables of the model, such as wages and price indexes, become random variables.

We begin by providing a general characterization of bilateral investments under uncertainty in terms of the first and second moments of the general equilibrium variables. This characterization holds under both complete and incomplete markets. All that differs is the values of these first and second moments of the general equilibrium variables. We solve the model using backward induction. We begin by characterizing the *ex post* equilibrium, given *ex ante* investments in import and export capacity, before turning to the determination of these *ex ante* investments below.

4.1 *Ex Post* Stage

After the *ex ante* investments in import and export capacity have been incurred, agents observe the *ex post* realizations for productivities and bilateral trade costs. The representative agent in importer n chooses consumption of the good produced by each exporter i to maximize utility, given these realizations for country productivities $\left\{z_i\left(\nu\right)\right\}_{i=1}^{N}$ and bilateral trade costs $\left\{\tau_{ni}\left(\nu\right)\right\}_{n=1,i=1}^{N,N}$, and the *ex ante* investments in import capacity $\{\iota_{ni}\}$ and export capacity $\{h_{ni}\}$:

$$\max_{\{c_{ni}(\nu)\}} \left\{ \left(\sum_{i=1}^{N} c_{ni}\left(\nu\right)^{\frac{\omega-1}{\omega}} \right)^{\frac{\omega}{\omega-1}} \right\}, \tag{11}$$

$$\text{s.t.} \qquad \sum_{i=1}^{N} p_{ni}^{c}(\nu)\, c_{ni}(\nu) = \mathcal{W}_n(\nu)\,. \tag{12}$$

where $p_{ni}^{c}(\nu)$ is the price of consuming the good supplied by exporter i in importer n (after investments in import and export capacity), which corresponds to the dual of equation (5) for given *ex ante* investments in import and export capacity.

From the first-order conditions to this problem, the representative agent's CES preferences (2) imply that each importer's relative expenditure shares across exporters are a constant elasticity function of relative prices:

$$\frac{p_{ni}^{c}(\nu)\, c_{ni}(\nu)}{\sum_{j=1}^{N} p_{nj}^{c}(\nu)\, c_{nj}(\nu)} = \frac{\left(p_{ni}^{c}(\nu)\right)^{1-\omega}}{\sum_{j=1}^{N}\left(p_{nj}^{c}(\nu)\right)^{1-\omega}} = \frac{c_{ni}(\nu)^{\frac{\omega-1}{\omega}}}{\sum_{j=1}^{N} c_{nj}(\nu)^{\frac{\omega-1}{\omega}}}. \tag{13}$$

The Cobb-Douglas trading technology (5) implies that bilateral expenditure on production labor ($w_i(\nu)\,\ell_{ni}(\nu)$) is a constant share of bilateral consumption expenditure ($p_{ni}^{c}(\nu)\, c_{ni}(\nu)$):

$$w_i(\nu)\,\ell_{ni}(\nu) = \delta p_{ni}^{c}(\nu)\, c_{ni}(\nu)\,. \tag{14}$$

This trading technology also implies that the *ex post* revenues of exporters and importers are constant shares of bilateral consumption expenditure. The resulting *ex post* quasi-rents of exporters ($\pi_{ni}^{h}(\nu)$) and importers ($\pi_{ni}^{\iota}(\nu)$), which can be either positive or negative, are rebated to the representative agent as follows:

$$\pi_{ni}^{h}(\nu) = \beta p_{ni}^{c}(\nu)\, c_{ni}(\nu) - w_i^{-} h_{ni},$$
$$\pi_{ni}^{\iota}(\nu) = \alpha p_{ni}^{c}(\nu)\, c_{ni}(\nu) - w_n^{-} \iota_{ni},$$

where w_n^{-} and w_i^{-} are the *ex ante* wages for labor employed in import capacity (ι_{ni}) and export capacity (h_{ni}), respectively; these wages and the bilateral investments do not depend on the state ν, because they are determined *ex ante*.

The system of *ex post* general equilibrium conditions equating the income of each exporter to expenditure on its goods now takes the following form:

$$w_i(\nu)\left(\bar{\ell}_i - \sum_{k=1}^{N}\iota_{ik} - \sum_{n=1}^{N} h_{ni}\right) = \sum_{n=1}^{N} \frac{\left(\frac{\iota_{ni}^{-\frac{\alpha}{\delta}} h_{ni}^{-\frac{\beta}{\delta}} \tau_{ni}(\nu) w_i(\nu)}{z_i(\nu)}\right)^{-\theta^{SR}}}{\sum_{j=1}^{N}\left(\frac{\iota_{nj}^{-\frac{\alpha}{\delta}} h_{nj}^{-\frac{\beta}{\delta}} \tau_{nj}(\nu) w_j(\nu)}{z_j(\nu)}\right)^{-\theta^{SR}}} \mathcal{W}_n^{y}(\nu), \tag{15}$$

where the *ex post* trade elasticity is $\theta^{SR} = \widetilde{\omega} - 1$ and $\widetilde{\omega} \equiv \frac{\omega}{\omega-\delta(\omega-1)}$; $\mathcal{W}_n^{y}(\nu)$ is *ex post* expenditure on traded goods, which is a constant fraction of aggregate *ex post* expenditure ($\mathcal{W}_n^{y}(\nu) = \delta\mathcal{W}_n(\nu)$); aggregate expenditure equals aggregate income under incomplete markets ($\mathcal{W}_n(\nu) = w_n(\nu)\,\bar{\ell}_n$); aggregate expenditure in general differs from aggregate income under complete markets, because of the state-contingent transfers ($\mathcal{W}_n(\nu) \neq w_n(\nu)\bar{\ell}_n$); we show in the next section that aggregate *ex ante* investments in import capacity ($\sum_{k=1}^{N} \iota_{ik}$) and export capacity ($\sum_{n=1}^{N} h_{ni}$) are constant fractions of the economy's labor endowment.

The *ex post* trade elasticity in this system of general equilibrium conditions corresponds to a *short-run* (superscript SR) elasticity of trade with respect to variable trade costs ($\theta^{SR} = \frac{\delta(\omega-1)}{(1-\delta)\omega+\delta}$), which is smaller in absolute value than the long-run elasticity in the previous section ($\theta^{SR} < \theta^{LR}$), because it does not incorporate adjustments in import and export capacity.[4]

This system of *ex post* general equilibrium conditions (15) takes a similar form to equation (9) in the deterministic Armington model, but with several differences. First, the *ex post* trade elasticity $\theta^{SR} = \frac{\delta}{(1-\delta)\omega+\delta}(\omega-1) < (\omega-1)$ is smaller in absolute value than in the deterministic Armington model, because traded goods are only part of consumption expenditure. Second, the *ex ante* investments in import and export capacity act like demand shifters in the *ex post* expenditure shares. These *ex ante* investments enter the *ex post* expenditure shares inversely to bilateral trade costs and can be absorbed into a composite measure of *ex post* adjusted bilateral trade costs: $\widetilde{\tau}_{ni}(\nu) \equiv \iota_{ni}^{-\alpha/\delta} h_{ni}^{-\beta/\delta} \tau_{ni}(\nu)$.

Third, labor used in production is only part of the economy's endowment, because of investments in import and export capacity. Fourth, *ex post* bilateral trade costs ($\widetilde{\tau}_{ni}(\nu) \equiv \iota_{ni}^{-\alpha/\delta} h_{ni}^{-\beta/\delta} \tau_{ni}(\nu)$) are now endogenously determined by the *ex ante* investments in import and export capacity, which depend on the economic environment (including uncertainty). Therefore, shocks to second moments can have first moment effects on the global economy, through endogenous changes in *ex post* bilateral trade costs, and hence bilateral trade shares.

4.2 *Ex Ante* Stage

In equilibrium, the representative agent must be indifferent between allocating labor *ex ante* to import and export capacity and allocating labor *ex post* to production for each trade partner. Therefore, the *ex ante* wage (w_i^-) weighted by the expected marginal utility of income must equal to the expectation of the wage times the marginal utility of income *ex post*:

$$\mathbb{E}\left[\frac{U'(\mathcal{C}_n(\nu))}{\mathcal{P}_n(\nu)}\right] w_n^- = \mathbb{E}\left[\frac{U'(\mathcal{C}_n(\nu))}{\mathcal{P}_n(\nu)} w_n(\nu)\right] \tag{16}$$

where the expectation $\mathbb{E}[\cdot]$ is taken across states (over the distribution of productivities and bilateral trade costs) and $\mathcal{P}_n(\nu)$ is the price index dual to consumption index (2) for a given state.

Importing firms choose *ex ante* investments to maximize profits, using the consumer's marginal utility of income as the stochastic discount factor, because all firms are owned by the representative

[4] For empirical evidence that trade elasticities are smaller in absolute value in the short-run than in the long-run, see for example Ruhl (2008), Anderson and Yotov (2020), and Boehm et al. (2023).

agent, and importer profits or losses are rebated back to the domestic consumer. Importing firms in country n choose import capacity (ι_{ni}) to solve the following problem for each export market i, taking prices and wages as given:

$$\max_{\iota_{ni}} \left\{ \mathbb{E}\left[\frac{\mathcal{U}'\left(\mathcal{C}_n\left(\nu\right)\right)}{\mathcal{P}_n\left(\nu\right)} \max_{x_{ni}(\nu)} \left(p^c_{ni}\left(\nu\right) c_{ni}\left(\nu\right) - p^x_{ni}\left(\nu\right) x_{ni}\left(\nu\right)\right)\right] - \mathbb{E}\left[\frac{U'\left(\mathcal{C}_n\left(\nu\right)\right)}{\mathcal{P}_n\left(\nu\right)}\right] w^-_n \iota_{ni} \right\}, \tag{17}$$

where $p^x_{ni}\left(\nu\right)$ is the price of the good supplied by exporter i to importer n on world markets (after investments in export capacity but before investments in import capacity), which corresponds to the dual of equation (4).

Exporting firms in country i choose export capacity (h_{ni}) to solve the following problem for each import market n, taking prices and wages as given:

$$\max_{h_{ni}} \left\{ \mathbb{E}\left[\frac{\mathcal{U}'\left(\mathcal{C}_i\left(\nu\right)\right)}{\mathcal{P}_i\left(\nu\right)} \max_{\ell_{ni}(\nu)} \left(p^x_{ni}\left(\nu\right) x_{ni}\left(\nu\right) - w_i\left(\nu\right) \ell_{ni}\left(\nu\right)\right)\right] - \mathbb{E}\left[\frac{U'\left(\mathcal{C}_i\left(\nu\right)\right)}{\mathcal{P}_i\left(\nu\right)}\right] w^-_i h_{ni} \right\}, \tag{18}$$

where the stochastic discount factors for the importer ($\mathcal{U}'\left(\mathcal{C}_n\left(\nu\right)\right)/\mathcal{P}_n\left(\nu\right)$) and exporter ($\mathcal{U}'\left(\mathcal{C}_i\left(\nu\right)\right)/\mathcal{P}_i\left(\nu\right)$) depend on whether markets are complete or incomplete.

4.3 Properties of the *Ex Ante* Equilibrium

Ex ante investments in import and export capacity are shaped by a number of forces. First, consumer love of variety ensures that importing and exporting firms disperse bilateral investments across trade partners. Second, there is an aggregate trade-off. The more labor that is allocated to import and export capacity *ex ante*, the less that is available for production *ex post*. Third, there is a bilateral trade-off. On the one hand, concentrating bilateral investments more towards the partner country with the best average characteristics raises expected real income. On the other hand, dispersing bilateral investments across a wider range of partner countries enhances the ability to import from partners that have worse average characteristics but that experience favorable realizations of these characteristics. From the first-order conditions to importing and exporting firms' profit maximization problems (17) and (18), we obtain the following proposition for these *ex ante* investments.

Proposition 1. ***(Ex Ante Equilibrium)*** *From profit maximization, zero profits and expenditure minimization, ex ante investments in the competitive equilibrium satisfy:*

$$\sum_{i=1}^{N} \iota_{ni} = \alpha \bar{\ell}_n, \quad \sum_{k=1}^{N} h_{ki} = \beta \bar{\ell}_i,$$

$$\iota_{ni} \propto_n \mathbb{E}\left[\frac{\mathcal{U}'\left(\mathcal{C}_n\left(\nu\right)\right)}{\mathcal{P}_n\left(\nu\right)} p^x_{ni}\left(\nu\right) x_{ni}\left(\nu\right)\right] = \mathbb{E}\left[\mathcal{C}_n\left(\nu\right)^{1-\gamma} \left(\frac{\iota_{ni}^{-\frac{\alpha}{\delta}} h_{ni}^{-\frac{\beta}{\delta}} \tau_{ni}\left(\nu\right) w_i\left(\nu\right)}{z_i\left(\nu\right)}\right)^{1-\widetilde{\omega}} \mathcal{Q}_n\left(\nu\right)^{-(1-\widetilde{\omega})}\right],$$

$$h_{ni} \propto_i \mathbb{E}\left[\frac{\mathcal{U}'(\mathcal{C}_i(\nu))}{\mathcal{P}_i(\nu)} p_{ni}^x(\nu) x_{ni}(\nu)\right] = \mathbb{E}\left[\frac{\mathcal{C}_i(\nu)^{1-\gamma}}{\mathcal{W}_i(\nu)} \left(\frac{\iota_{ni}^{-\frac{\alpha}{\delta}} h_{ni}^{-\frac{\beta}{\delta}} \tau_{ni}(\nu) w_i(\nu)}{z_i(\nu)}\right)^{1-\widetilde{\omega}} \mathcal{Q}_n(\nu)^{-(1-\widetilde{\omega})} \mathcal{W}_n(\nu)\right],$$

$$\mathcal{Q}_n(\nu) \equiv \left(\sum_{j=1}^{N} \left(\frac{\iota_{nj}^{-\frac{\alpha}{\delta}} h_{nj}^{-\frac{\beta}{\delta}} \tau_{nj}(\nu) w_j(\nu)}{z_j(\nu)}\right)^{1-\widetilde{\omega}}\right)^{\frac{1}{1-\widetilde{\omega}}},$$

$$\mathcal{C}_n(\nu) = \mathcal{Q}_n(\nu)^{-\delta} \mathcal{W}_n(\nu)^{\delta} \delta^{\delta}, \qquad \mathcal{P}_n(\nu) = \frac{\mathcal{W}_n(\nu)}{\mathcal{C}_n(\nu)} = \mathcal{Q}_n(\nu)^{\delta} \mathcal{W}_n(\nu)^{1-\delta} \delta^{-\delta},$$

where $\propto_n$ *means up to a multiplicative factor that depends only on* n *and* $\propto_i$ *means up to a multiplicative factor that depends only on* i.

Proof. See Online Appendices E.1 and E.2. □

Aggregate employments in import and export capacity in Proposition 1 are constant shares of each country's labor endowment. This property reflects the fact that import and export capacity are non-traded services that enter a trading technology (5) with constant cost shares. Without uncertainty, this property of constant cost shares implies that relative bilateral investments in import and export capacity are proportional to relative trade values ($p_{ni}^x x_{ni}$). With uncertainty, these relative bilateral investments in Proposition 1 are proportional to the relative expectations of trade values weighted by the stochastic discount factor for the importer or exporter. The terms for export revenues weighted by these stochastic discount factors inside the expectations in Proposition 1 can be expressed in terms of a CES *ex post* price index for traded goods ($\mathcal{Q}_n(\nu)$) that depends on *ex ante* investments in import and export capacity (ι_{ni}, h_{ni}); the wage rate in the exporter country ($w_i(\nu)$); and aggregate expenditure of the importer country ($\mathcal{W}_n(\nu)$).

The competitive market equilibrium coincides with the allocation that would be chosen by a collection of national planners who each maximized their representative agent's welfare, taking the *ex ante* distribution and *ex post* realization of international prices (the terms of trade) as given. This property reflects our assumption that all firms are owned by the representative agent, such that *ex post* profits and *ex ante* wages in the competitive equilibrium are weighted by the stochastic discount factor of the representative agent in the importer or exporter. Under incomplete markets, the competitive market allocation is inefficient, because the relative values of countries' stochastic discount factors differ across states. In contrast, under complete markets, the relative values of countries' stochastic discount factors are equalized across states.

Taking a second-order approximation in the expressions for *ex ante* investments in Proposition 1 around mean log productivities and trade costs ($\ln \boldsymbol{z} = \boldsymbol{\mu_z}, \ln \boldsymbol{\tau} = \boldsymbol{\mu_\tau}$), equilibrium *ex ante* investments solve the following general equilibrium portfolio problem, where to streamline notation we omit the implicit dependence of variables on state for the remainder of this subsection.

Proposition 2. ***(Import and Export Capacity)*** *Taking a second-order approximation around mean log productivities and trade costs* ($\ln \boldsymbol{z} = \boldsymbol{\mu_z}$, $\ln \boldsymbol{\tau} = \boldsymbol{\mu_\tau}$)*, ex ante investments satisfy:*

$$
\begin{aligned}
\iota_{ni} \propto_n \quad & \mathbb{E}\left[\left(\iota_{ni}^{-\frac{\alpha}{\delta}} h_{ni}^{-\frac{\beta}{\delta}} \frac{w_i \tau_{ni}}{z_i}\right)^{1-\widetilde{\omega}}\right] \\
& \times \exp\left(\mathit{Cov}\left(\ln \mathcal{C}_n^{-\gamma}, \ln\left(\frac{w_i \tau_{ni}}{z_i}\right)^{1-\widetilde{\omega}}\right)\right) \\
& \times \exp\left(\mathit{Cov}\left(\ln\left(\mathcal{W}_n/\mathcal{P}_n\right), \ln\left(\frac{w_i \tau_{ni}}{z_i}\right)^{1-\widetilde{\omega}}\right)\right) \\
& \times \exp\left(-\mathit{Cov}\left(\ln \mathcal{Q}_n^{1-\widetilde{\omega}}, \ln\left(\frac{w_i \tau_{ni}}{z_i}\right)^{1-\widetilde{\omega}}\right)\right),
\end{aligned}
\qquad
\begin{aligned}
h_{ni} \propto_i \quad & \mathbb{E}\left[\left(\iota_{ni}^{-\frac{\alpha}{\delta}} h_{ni}^{-\frac{\beta}{\delta}} \tau_{ni}\right)^{1-\widetilde{\omega}} \mathcal{Q}_n^{-(1-\widetilde{\omega})} \mathcal{W}_n\right] \\
& \times \exp\left(\mathit{Cov}\left(\ln \mathcal{C}_i^{-\gamma}, \ln\left(\frac{\tau_{ni}}{\mathcal{Q}_n}\right)^{1-\widetilde{\omega}} \mathcal{W}_n\right)\right) \\
& \times \exp\left(-\mathit{Cov}\left(\ln \mathcal{P}_i, \ln\left(\frac{\tau_{ni}}{\mathcal{Q}_n}\right)^{1-\widetilde{\omega}} \mathcal{W}_n\right)\right) \\
& \times \exp\left(\mathit{Cov}\left(\ln\left(\frac{w_i}{z_i}\right)^{1-\widetilde{\omega}}, \ln\left(\frac{\tau_{ni}}{\mathcal{Q}_n}\right)^{1-\widetilde{\omega}} \mathcal{W}_n\right)\right),
\end{aligned}
$$

where $\propto_n$ *means up to a multiplicative factor that depends only on* n*;* $\propto_i$ *means up to a multiplicative factor that depends only on* i*; and the expectation* ($\mathbb{E}[\cdot]$) *and covariance* ($\mathrm{Cov}(\cdot)$) *are taken across states (over the distribution of productivities and bilateral trade costs).*

Proof. See Online Appendices E.2.3 and E.2.4. □

From Proposition 2, importer n invests more in bilateral import capacity with exporter i if that exporter's goods are cheap (i) in expectation (first term); (ii) when importer n's marginal utility is high (second term); (iii) when importer n has high purchasing power (third term); (iv) when other goods in importer n's consumption bundle are expensive (fourth term). Similarly, exporter i invests more in bilateral export capacity with importer n if that importer has higher expenditure on good i (i) in expectation (first term); (ii) when exporter i's marginal utility is high (second term); (iii) when exporter i's consumption price index is low, such that revenue is valuable in real terms (third term); (iv) when exporter i's cost of production is low (fourth term).

In the absence of uncertainty, bilateral investments depend solely on the realized values of variables, which equal their expected values. However, even in this case, there is an incentive to disperse bilateral investments across trade partners, because of the consumer love of variety implied by the differentiation of goods across countries. Therefore, in contrast to conventional models of international finance, in which portfolio shares become indeterminate in the limiting case of no uncertainty (Devereux and Sutherland 2011), bilateral investments in our model remain determinate even without uncertainty because of consumer love of variety. More generally, in the presence of uncertainty, there is an additional incentive to disperse bilateral investments across trade partners, in order to be able to trade with partners that experience desirable realizations despite low expectations. As a result, bilateral investments, and hence bilateral trade costs and bilateral trade shares, depend not only on means, but also on variances and covariances.

An implication of this property is that the independence of irrelevant alternatives (IIA) no longer holds under uncertainty, despite our assumption of CES preferences. In the absence of uncertainty, the relative value of importer n's trade with two exporters i and j depends solely on the relative characteristics of those exporters, and is invariant with respect to the characteristics of other third exporters $m \notin \{i, j\}$. In contrast, in the presence of uncertainty, the relative value of importer n's trade with two exporters i and j depends on whether or not exporter $m \notin \{i, j\}$ is included in the choice set, since countries i and j can have different covariances with country m.

Although the expressions for bilateral investments (ι_{ni}, h_{ni}) in Proposition 2 hold under both complete and incomplete markets, the distribution of the general equilibrium variables, and hence the values of the variances and covariances, are different between these two different assumptions about asset trade. Nevertheless, under complete markets, there remains a trade-off between concentrating investments on countries with better average characteristics versus diversifying investments across countries, in order to be able to trade with countries with worse average characteristics but that experience favorable realizations of these characteristics.

Intuitively, *ex ante* trade in state-contingent claims under complete markets ensures that the ratios of countries' marginal utilities of income are equalized across states for any network of bilateral investments in import and export capacity. But the representative agent in each country is not indifferent across alternative networks of bilateral investments. She has an incentive to choose bilateral investments that raise certainty-equivalent welfare, by for example reducing trade costs with countries whose marginal production costs are negatively correlated with those of other countries.

Proposition 2 holds up to a second-order approximation for any distribution of productivities and bilateral trade costs. The covariances that shape bilateral investments in this proposition involve endogenous variables that are determined in general equilibrium, including the consumption index ($\mathcal{C}_n$), the consumption price index ($\mathcal{P}_n$), the traded goods price index ($\mathcal{Q}_n$), and wages (w_n). We now provide an analytical characterization of the first and second moments of the model's general equilibrium variables, which depends upon whether or not there is trade in financial assets.

5 Uncertainty and Incomplete Markets

We begin by undertaking this characterization of the properties of the distribution of the general equilibrium variables under financial autarky, motivated by empirical evidence of relatively little consumption risk sharing across countries.[5] We exogenously set state-contingent transfers to zero, such that expenditure equals income ($\mathcal{W}_n(\nu) = w_n(\nu)\,\bar{\ell}_n$), and trade is balanced, in each state of the world. We choose world GDP in each state as the numeraire: $\sum_{i=1}^{N} w_i(\nu)\,\bar{\ell}_i = 1$. In Section 6 below, we provide an analogous characterization with state-contingent transfers, and demonstrate similar predictions for the impact of uncertainty on patterns of trade, income and welfare. Throughout the remainder of this section, we suppress the implicit dependence of variables on state to streamline notation, except where otherwise indicated.

[5] See Coeurdacier and Rey (2013) for a review of this evidence on consumption-risk sharing across countries.

5.1 Distribution of General Equilibrium Variables

To simplify the exposition, we begin by assuming that bilateral trade costs $\{\tau_{ni}\}_{n=1,i=1}^{N,N}$ are deterministic, such that only productivities $\{z_i\}_{i=1}^{N}$ are stochastic. In Proposition 3, we take a second-order Taylor-series expansion in the income accounting condition (15) around mean log productivity in each country. We provide an analytical characterization of the first and second moments of the wage distribution, from which the first and second moments of all other endogenous variables can be recovered. In Proposition 5 below, we generalize these results to allow both productivities $\{z_i\}_{i=1}^{N}$ and bilateral trade costs $\{\tau_{ni}\}_{n=1,i=1}^{N,N}$ to be stochastic.

Proposition 3. ***(Incomplete Markets Wage Jacobian and Hessian for Productivity Uncertainty)*** *Define $\boldsymbol{F}$ as the equilibrium function (equation (15)) that maps log productivities ($\ln \boldsymbol{z}$) into log wages ($\ln \boldsymbol{w}$) given the trade elasticity (θ): $\ln \boldsymbol{w} = \boldsymbol{F}(\ln \boldsymbol{z}; \theta)$. This equilibrium function is isomorphic to that in the deterministic Armington model in equation (9) up to adjustments to the trade elasticity (θ) and bilateral trade costs ($\widetilde{\tau}_{ni} \equiv \iota_{ni}^{-\alpha/\delta} h_{ni}^{-\beta/\delta} \tau_{ni}$).*
(A) *Up to second order, the first and second moments of log wages satisfy:*

$$\mathbb{E}\left[\ln w_i\right] \approx F_i\left(\boldsymbol{\mu}_z\right) + \frac{1}{2}\sum_{j=1}^{N}\sum_{k=1}^{N} \boldsymbol{H}_{jk}^{\boldsymbol{w}i}\left[\boldsymbol{\Sigma}_z\right]_{jk}, \qquad \mathbb{V}\left[\ln \boldsymbol{w}\right] \approx \boldsymbol{J}^{\boldsymbol{w}}\boldsymbol{\Sigma}_z\boldsymbol{J}^{\boldsymbol{w}\prime}, \tag{19}$$

where $\mathbb{V}[\cdot]$ denotes the variance operator and $\boldsymbol{J}^{\boldsymbol{w}}$ and $\boldsymbol{H}^{\boldsymbol{w}}$ are the Jacobian and Hessian of $\boldsymbol{F}(\ln \boldsymbol{z}; \theta) = \ln \boldsymbol{w}$ evaluated at $\ln \boldsymbol{z} = \boldsymbol{\mu}_z$.
(B) *The bilateral expenditure share matrix ($\boldsymbol{S}$) and income vector ($\boldsymbol{q}$) are sufficient statistics for the Jacobian and Hessian:*

$$S_{ni} \equiv \left.\frac{\left(\widetilde{\tau}_{ni} w_i / z_i\right)^{-\theta}}{\sum_{k=1}^{N}\left(\widetilde{\tau}_{nk} w_k / z_k\right)^{-\theta}}\right|_{\ln \boldsymbol{z} = \boldsymbol{\mu}_z}, \qquad T_{in} \equiv \left.\frac{w_n \bar{\ell}_n S_{ni}}{w_i \bar{\ell}_i}\right|_{\ln \boldsymbol{z} = \boldsymbol{\mu}_z}.$$

$$\boldsymbol{J}^{\boldsymbol{w}} \equiv \left[\frac{\mathrm{d}\ln w_i}{\mathrm{d}\ln z_j}\Big|_{\ln \boldsymbol{z} = \boldsymbol{\mu}_z}\right] = -\theta \boldsymbol{X}^{-1}\left(\boldsymbol{T}\boldsymbol{S} - \boldsymbol{I}\right), \quad \boldsymbol{X} \equiv \boldsymbol{I} - \boldsymbol{T} - \theta\left(\boldsymbol{T}\boldsymbol{S} - \boldsymbol{I}\right) + (1+\theta)\boldsymbol{Q},$$

$$\boldsymbol{H}_{jk}^{\boldsymbol{w}i} \equiv \partial_k \boldsymbol{J}_{ij}^{\boldsymbol{w}}, \qquad \partial_k \boldsymbol{J}^{\boldsymbol{w}} = \left(\boldsymbol{I} - \boldsymbol{Q}\right)\left\{\boldsymbol{X}^{-1}\left(\partial_k \boldsymbol{T}\right)\left(\left(\boldsymbol{I} + \theta \boldsymbol{S}\right)\boldsymbol{J}^{\boldsymbol{w}} - \theta \boldsymbol{S}\right) + \theta \boldsymbol{X}^{-1}\boldsymbol{T}\left(\partial_k \boldsymbol{S}\right)\left(\boldsymbol{J}^{\boldsymbol{w}} - \boldsymbol{I}\right)\right\},$$

$$\left(\partial_k \boldsymbol{S}\right)_{ni} = \theta \boldsymbol{S}_{ni}\left(\mathbf{1}_{i=k} - \boldsymbol{J}_{ik}^{\boldsymbol{w}} - \boldsymbol{S}_{nk} + \left(\boldsymbol{S}\boldsymbol{J}^{\boldsymbol{w}}\right)_{nk}\right),$$

$$\left(\partial_k \boldsymbol{T}\right)_{in} = \boldsymbol{T}_{in}\left[\boldsymbol{J}_{nk}^{\boldsymbol{w}} - \boldsymbol{J}_{ik}^{\boldsymbol{w}} + \theta\left(\mathbf{1}_{i=k} - \boldsymbol{J}_{ik}^{\boldsymbol{w}} - \boldsymbol{S}_{nk} + \left(\boldsymbol{S}\boldsymbol{J}^{\boldsymbol{w}}\right)_{nk}\right)\right].$$

The matrix $\boldsymbol{Q} \equiv \mathbf{1}\boldsymbol{q}'$ captures our choice of world GDP as the numeraire.

Proof. See Online Appendix F.2. □

Proposition 3 characterizes the first and second moments of wages in terms of the sufficient statistics of income shares ($q_i = w_i \bar{\ell}_i$) and expenditure shares (S_{ni}) and the trade elasticity (θ).

From these sufficient statistics of country incomes ($q_i = w_i\bar{\ell}_i$) and expenditure shares (s_{ni}), we can solve for income shares ($t_{ik} = s_{ki}w_k\bar{\ell}_k/w_i\bar{\ell}_i$). Part (A) of the proposition carries over unchanged to complete markets, whereas the formulas in Part (B) change, but again depend on sufficient statistics and the trade elasticity.

The elements of the wage Jacobian ($\boldsymbol{J}^{\boldsymbol{w}}_{in}$) in Proposition 3 capture the general equilibrium elasticity of the wage in each country i with respect to productivity in each country n ($\mathrm{d}\ln w_i/\mathrm{d}\ln z_n$) in the income accounting condition (15). These general equilibrium elasticities depend on the share of income ($t_{ik} = s_{ki}w_k\bar{\ell}_k/w_i\bar{\ell}_i$) that country i derives from each market k and the share of expenditure that market k allocates to country n (s_{kn}). An increase in productivity in country n, which raises the competitiveness of its goods, tends to reduce the wage in country i through cross-substitution effects if country i obtains large shares of its income from markets in which country n accounts for large shares of expenditure. The elements of the wage Hessian ($\boldsymbol{H}^{\boldsymbol{w}i}_{nk}$) in Proposition 3 capture the sensitivity of these general equilibrium wage elasticities for country i and country n with respect to productivity in country k ($\mathrm{d}^2\ln w_i/\mathrm{d}\ln z_n \mathrm{d}\ln z_k$).

The system of general equilibrium conditions for wages (15) in our model of import and export capacity is isomorphic to that in the deterministic Armington model in equation (9), up to adjustments to the trade elasticity and bilateral trade costs. This isomorphism implies that our results for the wage Jacobian and Hessian in Proposition 3 hold for the deterministic Armington model and the wider class of constant elasticity quantitative trade models. Therefore, our results can be used in any applications of these models to uncertainty.

In our empirical application, we solve bilateral trade costs and productivities by inverting the general equilibrium conditions of our model, given observed data on bilateral trade ($X_{ni} = p^x_{ni}x_{ni}$), income (q_n) and price indexes ($\mathcal{P}_n$). This model inversion takes into account that productivity is uncertain and that bilateral investments in import and export capacity depend on the first and second moments of the endogenous variables. From this model inversion, we recover time-invariant bilateral trade costs for a benchmark year and time-varying country productivities, which we use to compute the mean ($\boldsymbol{\mu_z}$) and variance-covariance matrix ($\boldsymbol{\Sigma_z}$) of country log productivities. Given these solutions for the first and second moments of log productivities and the matrix of bilateral trade costs, we solve for equilibrium incomes (q_i), expenditure shares (S_{ni}) and income shares (T_{in}) at mean log productivities ($\boldsymbol{\mu_z}$), and evaluate the wage Jacobian ($\boldsymbol{J}^{\boldsymbol{w}}$) and Hessian ($\boldsymbol{H}^{\boldsymbol{w}}$) using Proposition 3.

Proposition 3 focuses on wages, because general equilibrium in our model reduces to the system of income accounting relationships that equate each country's income to expenditure on its goods. But the first and second moments of all other endogenous variables can be written in the same form as in equation (19) in terms of a Jacobian and Hessian, where the elements of these Jacobians and Hessians depend on the sufficient statistics of income shares ($q_i = w_i\bar{\ell}_i$) and expenditure shares

(S_{ni}) and the trade elasticity (θ). We use these results for the Jacobian and Hessians of the other endogenous variables to compute the covariance terms in the portfolio choice for import and export capacity in Proposition 2. The general equilibrium of our model corresponds to a fixed point in which the *ex ante* investments in Proposition 2 based on conjectured values of the first and second moments of the endogenous variables imply distributions of the endogenous variables that are consistent with these conjectures in Proposition 3.

Proposition 3 holds up to a second-order approximation regardless of the functional form of the productivity distribution (as long as it has well-defined first and second moments). But an alternative interpretation can be provided to the results in Proposition 3 based on a well-known result in the trade and finance literatures that a weighted sum of log normally distributed random variables is approximately log normally distributed (Campbell and Viceira 2001 and Allen and Atkin 2022). Taking logarithms in the income accounting equation (15), our model implies that log wages can be expressed as the log of a weighted sum of wages, where the weights themselves depend on productivities and wages. If productivities are joint log normally distributed, Online Appendix H shows that wages are approximately joint log normally distributed. Therefore, this alternative interpretation provides a reason why the wage distribution should be approximately joint log normally distributed if productivity itself is joint log normally distributed, such that the wage distribution is well approximated by its first and second moments.

Propositions 2 and 3 characterize the general equilibrium of our model up to second-order and allow us to derive sharp analytical results of the effect of uncertainty on trade patterns, income and welfare, without imposing an assumed functional form for the productivity distribution. This analytical characterization reveals that bilateral investments solve a mean-variance problem in general equilibrium and provides the relevant sufficient statistics for computing the means, variances and covariances of the endogenous variables. We obtain these analytical results despite our multi-country quantitative setting, with many asymmetric countries, a rich geography of bilateral trade costs, and a general productivity distribution. Otherwise, characterizing the impact of uncertainty in quantitative trade models is intractable, because one can only solve the non-linear model numerically for a particular set of productivity realizations, and simulate the distribution of endogenous variables under an assumed functional form for the productivity distribution.

In Online Appendix L, we compare our analytical characterization of the first and second moments of the model's endogenous variables to simulated first and second moments from solving the non-linear model numerically under an assumed log normal productivity distribution. Consistent with results in the macroeconomics literature that perturbation methods provide a close approximation to the solution of non-linear heterogeneous agent models (e.g., Bhandari et al. 2023; Auclert et al. 2024), we find that our analytical first and second moments provide a close approximation to the simulated first and second moments under this assumed productivity distribution.

5.2 Welfare Gains from Trade

In a deterministic constant elasticity trade model, the opening of the closed economy to trade raises welfare through a higher level of real income (first moment). In the presence of uncertainty, the opening of trade also affects welfare through the volatility of real income (second moment). To evaluate these effects, we define certainty-equivalent welfare as the deterministic level of welfare that is equal to the expected value of uncertain welfare:

$$\mathbb{U}_n \equiv \mathcal{U}^{-1}\left(\mathbb{E}\left[\mathcal{U}\left(\mathcal{C}_n\left(\nu\right)\right)\right]\right) = \mathbb{E}\left[\mathcal{C}_n\left(\nu\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}. \tag{20}$$

To a second-order approximation, the log of certainty-equivalent welfare can be expressed in terms of the mean and variance of the log consumption index:

$$\ln \mathbb{U}_n \approx \mathbb{E}\left[\ln \mathcal{C}_n\left(\nu\right)\right] + \frac{1-\gamma}{2}\mathbb{V}\left[\ln \mathcal{C}_n\left(\nu\right)\right]. \tag{21}$$

Taking log differences between the open and closed-economy equilibria, the welfare gains from opening the closed economy to trade can be expressed up to second order as:

$$\begin{aligned} \ln\left[\mathbb{U}_n^O/\mathbb{U}_n^A\right] &\approx \mathbb{E}\left[\ln \mathcal{C}_n^O\left(\nu\right) - \ln \mathcal{C}_n^A\left(\nu\right)\right] + \frac{1-\gamma}{2}\left(\mathbb{V}\left[\ln \mathcal{C}_n^O\left(\nu\right)\right] - \mathbb{V}\left[\ln \mathcal{C}_n^A\left(\nu\right)\right]\right), \quad (22) \\ &= \mathbb{E}\left[\alpha \ln\left(\frac{\iota_{nn}^O}{\alpha \bar{\ell}_n}\right) + \beta \ln\left(\frac{h_{nn}^O}{\beta \bar{\ell}_n}\right) + \delta \ln\left(s_{nn}^O\left(\nu\right)\right)^{\frac{1}{1-\tilde{\omega}}}\right] \\ &\quad + \frac{1-\gamma}{2}\left(\mathbb{V}\left[\ln\left(w_n^O\left(\nu\right)/\mathcal{P}_n^O\left(\nu\right)\right)\right] - \delta^2 \mathbb{V}\left[\ln z_n\left(\nu\right)\right]\right), \end{aligned}$$

where the equation on the second line uses $\mathcal{C}_n\left(\nu\right) = \delta^\delta\left(\mathcal{W}_n\left(\nu\right)/\mathcal{Q}_n\left(\nu\right)\right)^\delta$ from Proposition 1 and the equality between aggregate expenditure and aggregate income under incomplete markets ($\mathcal{W}_n\left(\nu\right) = w_n\left(\nu\right)\bar{\ell}_n$).

In the deterministic Armington model, the welfare gains from opening the closed-economy to trade are summarized by the ACR formula in terms of the open-economy domestic expenditure share (s_{nn}^O) and the trade elasticity ($\theta = \omega - 1$). In our model of aggregate uncertainty, a version of that formula holds for the first moment effects of trade (second line in equation (22)), with two modifications. First, the trade elasticity is no longer the elasticity of substitution between country varieties, but instead also depends on the expenditure shares of import and export capacity ($\theta = \frac{\delta(\omega-1)}{(1-\delta)\omega+\delta}$, where recall $\delta = 1-\alpha-\beta$). Second, domestic investments in import capacity and export capacity are different in the closed and open economies, which implies that the ACR formula must be adjusted to take account of these differences. This adjustment implies that the first moment effects of trade on welfare are smaller in our model than in the ACR formula (since $\iota_{nn}^O < \alpha\bar{\ell}_n$ and $h_{nn}^O < \beta\bar{\ell}_n$ in equation (22)). Intuitively, the closed economy is less bad than implied by the observed open-economy domestic expenditure share, because domestic investments would increase in the closed economy to reduce the costs of the economy trading with itself.

In our model of aggregate uncertainty, the welfare effects from opening the closed economy to trade are further modified by changes in the second moment of real income (third line in equation (22)). In the closed economy, the variance in log consumption is proportional to the variance in domestic productivity ($\mathbb{V}\left[\ln \mathcal{C}_n^A(\nu)\right] = \delta^2 \mathbb{V}\left[\ln z_n(\nu)\right]$). In contrast, in the open economy, the variance in log consumption ($\mathbb{V}\left[\ln \mathcal{C}_n^O(\nu)\right] = \mathbb{V}\left[\ln\left(w_n^O(\nu)/\mathcal{P}_n^O(\nu)\right)\right]$) depends on the covariance structure of domestic and foreign shocks. As a result, the volatility of real income can be either higher or lower in the open economy than in the closed economy. Furthermore, some countries can experience an increase in the volatility of their real income from the opening of trade, while others experience a decrease in the volatility of their real income.

Nevertheless, we noted above that the competitive equilibrium of our economy coincides with the solution to the problem of a collection of national planners, who each maximize national welfare taking the *ex ante* distribution and *ex post* realization of international prices (the terms of trade) as given. Since these national planners choose positive investments with foreign trade partners in the open economy (when they could choose zero investments with foreign trade partners and induce autarky), revealed preference implies that welfare is weakly higher in the open economy than the closed economy.

Proposition 4. ***(Welfare Gains from Trade)*** *Ex ante welfare is weakly higher in the open economy than in the closed economy, regardless of whether the opening of trade raises or reduces the volatility of real income.*

Proof. See Online Appendix F.4. □

5.3 Stochastic Trade Costs

We now extend Proposition 3 to incorporate stochastic trade costs. Let $\boldsymbol{\Sigma}$ denote the $K \times K$ covariance matrix of K mean-zero random variables (factors) $\{\epsilon_k\}_{k=1}^K$. We use this factor representation to allow for dimension reduction ($K \leq N + N^2$), but doing so is without loss of generality, because we incorporate the possibility of as many factors as random variables. Let $\boldsymbol{B_z}$ denote the $N \times K$ matrix capturing the factor loading of log productivities, so that log-productivity $\ln z_i$ follows:

$$\ln z_i = \mu_{z_i} + \sum_{k=1}^{K} \left[\boldsymbol{B_z}\right]_{i,k} \epsilon_k, \tag{23}$$

Likewise, let $\boldsymbol{B_\tau}$ denote the $N^2 \times K$ matrix capturing the factor loading of log bilateral trade costs. The covariance matrix of productivities and trade costs are, respectively,

$$\boldsymbol{\Sigma_z} = \boldsymbol{B_z} \boldsymbol{\Sigma} \boldsymbol{B_z'}, \qquad \boldsymbol{\Sigma_\tau} = \boldsymbol{B_\tau} \boldsymbol{\Sigma} \boldsymbol{B_\tau'}. \tag{24}$$

We also have that $Cov\left(\ln z_i, \ln \tau_{mj}\right)$ is the (i, mj)-th entry of $\boldsymbol{\Sigma_{z\tau}} \equiv \boldsymbol{B_z} \boldsymbol{\Sigma} \boldsymbol{B_\tau'}$. Using this factor representation, we have the following result.

Proposition 5. ***(Incomplete Markets Wage Jacobian and Hessian for Productivity and Trade Cost Uncertainty)*** *Define $\boldsymbol{F}$ as the equilibrium function in equation (15) that maps log productivities ($\ln \boldsymbol{z}$) and log trade costs ($\ln \boldsymbol{\tau}$) into log wages ($\ln \boldsymbol{w}$) given the trade elasticity (θ): $\ln \boldsymbol{w} = \boldsymbol{F}(\ln \boldsymbol{z}, \ln \boldsymbol{\tau}; \theta)$. This equilibrium function is isomorphic to that in the deterministic Armington model in equation (9) up to an adjustment to the trade elasticity (θ) and bilateral trade costs ($\widetilde{\tau}_{ni} \equiv \iota_{ni}^{-\alpha/\delta} h_{ni}^{-\beta/\delta} \tau_{ni}$).*

(A) *Up to second order, the first and second moments of log wages satisfy:*

$$\mathbb{E}[\ln w_k] \approx F_k(\boldsymbol{\mu}_z, \boldsymbol{\mu}_\tau) + \frac{1}{2}\sum_{ni,mj} \frac{\mathrm{d}^2 \boldsymbol{F}_k}{\mathrm{d}\ln \tau_{ni}\, \mathrm{d}\ln \tau_{mj}} [\boldsymbol{\Sigma}_\tau]_{ni,mj} + \frac{1}{2}\sum_{i,j} \frac{\mathrm{d}^2 \boldsymbol{F}_k}{\mathrm{d}\ln z_i\, \mathrm{d}\ln z_j} [\boldsymbol{\Sigma}_z]_{i,j} + \sum_{i,mj} \frac{\mathrm{d}^2 \boldsymbol{F}_k}{\mathrm{d}\ln z_i\, \mathrm{d}\ln \tau_{mj}} [\boldsymbol{\Sigma}_{z\tau}]_{i,mj}$$

$$V[\ln \boldsymbol{w}] \approx \boldsymbol{J}_{\boldsymbol{z}}^{\boldsymbol{w}} \Sigma_z \boldsymbol{J}_{\boldsymbol{z}}^{\boldsymbol{w}\prime} + \boldsymbol{J}_{\boldsymbol{\tau}}^{\boldsymbol{w}} \Sigma_\tau \boldsymbol{J}_{\boldsymbol{\tau}}^{\boldsymbol{w}\prime} + \boldsymbol{J}_{\boldsymbol{z}}^{\boldsymbol{w}} \Sigma_{z\tau} \boldsymbol{J}_{\boldsymbol{\tau}}^{\boldsymbol{w}\prime} + \boldsymbol{J}_{\boldsymbol{\tau}}^{\boldsymbol{w}} \Sigma_{z\tau}' \boldsymbol{J}_{\boldsymbol{z}}^{\boldsymbol{w}\prime}$$

where the Jacobian matrices $\boldsymbol{J}_{\boldsymbol{z}}^{\boldsymbol{w}} \equiv \left[\frac{\mathrm{d}\ln w_k}{\mathrm{d}\ln z_i}\right]$ and $\boldsymbol{J}_{\boldsymbol{\tau}}^{\boldsymbol{w}} \equiv \left[\frac{\mathrm{d}\ln w_k}{\mathrm{d}\ln \tau_{ni}}\right]$ and the Hessians (the second derivatives) are evaluated at $(\ln \boldsymbol{z}, \ln \boldsymbol{\tau}) = (\boldsymbol{\mu}_z, \boldsymbol{\mu}_\tau)$.

(B) *The bilateral expenditure share matrix ($\boldsymbol{S}$) and income vector ($\boldsymbol{q}$) are sufficient statistics for the Jacobians and Hessians.*

Proof. See the Online Appendix F.5. □

Proposition 5 again characterizes the first and second moments of wages in terms of sufficient statistics and the trade elasticity. Therefore, we can also use our approach to study the effects of changes in the covariance structure of bilateral trade costs. Our framework highlights that changes in the second moment of bilateral trade costs (e.g., transportation disruptions and uncertainty over transportation costs) affect the first moment of bilateral investments in import and export capacity, and hence the first moments of bilateral trade, income and welfare. We focus here on bilateral trade costs, but analogous results hold for tariffs (incorporating changes in tariff revenue).

6 Uncertainty and Complete Markets

We next characterize the properties of the distribution of the general equilibrium variables under the alternative assumption of a full set of state-contingent Arrow-Debreu securities (complete markets). Recall that we index states of the world by ν, where a state corresponds to a set of realizations of country productivities $\{z_i(\nu)\}_{i=1}^N$ and bilateral trade costs $\{\tau_{ni}(\nu)\}_{n=1,i=1}^{N,N}$. We denote the probability density function of states by $g(\nu)$. We assume that agents can trade a full set of state-contingent Arrow-Debreu securities *ex ante*, before observing the realizations for productivities $\{z_i(\nu)\}_{i=1}^N$ and bilateral trade costs $\{\tau_{ni}(\nu)\}_{n=1,i=1}^{N,N}$. Each Arrow-Debreu security purchased by country n pays a unit of consumption to that country in state ν. We denote the *ex ante* price of this Arrow-Debreu security by $r(\nu)$.

6.1 Equilibrium Consumption

We have already characterized *ex post* consumption in terms of aggregate expenditure ($\mathcal{W}_n(\nu)$) given the realizations for productivities $\{z_i(\nu)\}_{i=1}^{N}$ and trade costs $\{\tau_{ni}(\nu)\}_{n=1,i=1}^{N,N}$ in Proposition 1. Under complete markets, aggregate expenditure ($\mathcal{W}_n(\nu)$) differs from wage income ($w_n(\nu)\,\bar{\ell}_n$) in each state ν, because of the state-contingent transfers. The representative agent chooses consumption in each state ($\mathcal{C}_n(\nu)$) to maximize *ex ante* expected utility subject to their budget constraint:

$$\max_{\{C_n(\nu)\}} \quad \int_{\nu} \frac{\mathcal{C}_n(\nu)^{1-\gamma}}{1-\gamma} g(\nu)\mathrm{d}\nu, \tag{25}$$

$$\text{s.t.} \int_{\nu} r(\nu)\left(\mathcal{P}_n(\nu)\mathcal{C}_n(\nu) - w_n(\nu)\,\bar{\ell}_n\right)\mathrm{d}\nu = 0.$$

From the first-order conditions to this problem, equilibrium consumption solves:

$$\frac{\mathcal{C}_n(\nu)^{-\gamma}}{\mathcal{P}_n(\nu)} = \lambda_n \underbrace{\frac{r(\nu)}{g(\nu)}}_{\equiv m(\nu)}, \tag{26}$$

where λ_n is the Lagrange multiplier and we have defined $m(\nu)$ as the pricing kernel.

Noting that the Lagrange multiplier (λ_n) is common across states, the share of expenditure allocated to state ν is:

$$\frac{\mathcal{P}_n(\nu)\mathcal{C}_n(\nu)}{\mathbb{E}[\mathcal{P}_n(\nu')\mathcal{C}_n(\nu')]} = \frac{m(\nu)^{-1/\gamma}\mathcal{P}_n(\nu)^{1-1/\gamma}}{\mathbb{E}[m(\nu')^{-1/\gamma}\mathcal{P}_n(\nu')^{1-1/\gamma}]}, \tag{27}$$

where $\mathbb{E}[\cdot]$ denotes the expectations operator across states (over the distributions of productivities and bilateral trade costs). A natural choice for the numeraire in each state is the pricing kernel $m(\nu) = 1$. Using this choice of numeraire, income net of Arrow-Debreu transfers (which equals expenditure) in state ν is:

$$\mathcal{W}_n(\nu) = \mathcal{P}_n(\nu)\mathcal{C}_n(\nu) = \underbrace{\frac{\mathbb{E}[w_n(\nu')\,\bar{\ell}_n]}{\mathbb{E}[\mathcal{P}_n(\nu')^{1-1/\gamma}]}}_{\equiv \Lambda_n} \mathcal{P}_n(\nu)^{1-1/\gamma}, \tag{28}$$

where Λ_n corresponds to country n's state-independent wealth (permanent-income), which scales its entire state-contingent consumption and price profile across states.

Note that the equilibrium condition (28) pins down relative prices across states, but does not pin down the overall price level in each state, because it is homogeneous of degree zero in the consumption price index ($\mathcal{P}_n$). To pin down the overall price level, we normalize world gross domestic product (GDP) to one when log productivities and log bilateral trade costs take their mean values:

$$\sum_{i=1}^{N} w_i(\bar{\nu})\,\bar{\ell}_i = 1, \tag{29}$$

where $\overline{\nu}$ denotes the state in which log productivities and log bilateral trade costs take their mean values $(\boldsymbol{\mu_z}, \boldsymbol{\mu_\tau})$. This normalization under complete markets corresponds to our choice of numeraire under incomplete markets, thereby allowing for meaningful comparisons between these two cases.

6.2 Distribution of General Equilibrium Variables

Income accounting in the *ex post* equilibrium requires that labor income in each country in each state ν $(w_i(\nu)\,\overline{\ell}_i)$ equals expenditure on the goods produced by that country, where aggregate expenditure in each country $(\mathcal{W}_n(\nu))$ now includes the state-contingent transfers:

$$w_i(\nu)\,\overline{\ell}_i = \sum_{n=1}^{N} s_{ni}(\nu)\,\frac{\mathcal{P}_n(\nu)^{1-1/\gamma}}{\mathbb{E}\left[\mathcal{P}_n(\nu')^{1-1/\gamma}\right]}\mathbb{E}\left[w_n(\nu')\right]\overline{\ell}_n, \tag{30}$$

where we used our solution for expenditure from equation (28).

We now use this income accounting condition (30) to characterize the first and second moments of the general equilibrium variables. For the remainder of this section, we suppress the implicit dependence of variables on state in order to streamline notation, except where otherwise indicated. As under incomplete markets, we assume that bilateral trade costs $\{\tau_{ni}\}_{n=1,i=1}^{N,N}$ are deterministic, such that only productivities $\{z_i\}_{i=1}^{N}$ are stochastic. In Proposition 6, we take a second-order Taylor-series expansion of the income accounting condition (30) around mean log productivity in each country.

Proposition 6. ***(Complete Markets Wage Jacobian and Hessian for Productivity Uncertainty)*** *Define $\boldsymbol{F}$ as the equilibrium function in equation (30) that maps log productivities to log wages as $\ln \mathbf{w} = F(\ln \boldsymbol{z}; \boldsymbol{\mu_z}, \boldsymbol{\Sigma_z}, \boldsymbol{\mu_w}, \boldsymbol{\Sigma_w})$, where we make explicit that the realized wage under complete markets depends on the moments of the wage and productivity distributions $(\boldsymbol{\mu_z}, \boldsymbol{\Sigma_z}, \boldsymbol{\mu_w}, \boldsymbol{\Sigma_w})$ as well as the realizations of productivities $(\boldsymbol{z})$.*

(A) *To second order,*

$$\mathbb{E}\left[\ln w_i\right] \approx F_i(\boldsymbol{\mu_z}) + \frac{1}{2}\sum_{j,k}\left(\boldsymbol{H}^{\boldsymbol{w}i}_{jk}\left(\boldsymbol{\Sigma_z}\right)_{jk}\right) \tag{31}$$

$$\mathbb{V}\left[\ln \boldsymbol{w}\right] \approx \boldsymbol{J^w}\boldsymbol{\Sigma_z}\boldsymbol{J^{w\prime}} \tag{32}$$

where $\mathbb{V}[\cdot]$ denotes the variance operator; the wage Jacobian $(\boldsymbol{J^w})$ and the wage Hessian $(\boldsymbol{H^w} \equiv \partial_k \boldsymbol{J^w})$ are themselves functions of the moments of the wage distributions $(\boldsymbol{\mu_w}, \boldsymbol{\Sigma_w})$.

(B) *The bilateral expenditure share matrix $(\boldsymbol{S})$ and income vector $(\boldsymbol{q})$ are sufficient statistics for the Jacobian and Hessian:*

$$S_{ni} \equiv \left.\frac{\left(\widetilde{\tau}_{ni}w_i/z_i\right)^{-\theta}}{\sum_{k=1}^{N}\left(\widetilde{\tau}_{nk}w_k/z_k\right)^{-\theta}}\right|_{\ln \boldsymbol{z}=\boldsymbol{\mu}_z}, \qquad T_{in} \equiv \left.\frac{\mathcal{W}_n S_{ni}}{w_i\overline{\ell}_i}\right|_{\ln \boldsymbol{z}=\boldsymbol{\mu}_z}, \qquad \widetilde{\tau}_{ni} \equiv \iota_{ni}^{-\alpha/\delta}h_{ni}^{-\beta/\delta}\tau_{ni}.$$

$$\boldsymbol{J^w} \equiv \left[\frac{\mathrm{d}\ln w_i}{\mathrm{d}\ln z_j}\Big|_{\ln \boldsymbol{z}=\boldsymbol{\mu}_z}\right] = -\left(\boldsymbol{I}-\boldsymbol{Q}\right)\theta\boldsymbol{X}^{-1}\left(\vartheta\boldsymbol{T}\boldsymbol{S}-\boldsymbol{I}\right) = \left(\boldsymbol{I}-\boldsymbol{Q}\right)\left(\boldsymbol{I}-\boldsymbol{X}^{-1}\right),$$

$$\boldsymbol{X} \equiv \boldsymbol{I} - \theta(\vartheta \boldsymbol{T}\boldsymbol{S} - \boldsymbol{I}),$$

$$\vartheta \equiv 1 + \frac{\delta(1-1/\gamma)}{\theta\kappa}, \qquad \kappa \equiv \delta + \frac{1-\delta}{\gamma} = 1 - (1-\delta)\big(1 - \tfrac{1}{\gamma}\big),$$

$$\boldsymbol{H}^{\boldsymbol{w}_i}_{jk} \equiv \partial_k \boldsymbol{J}^{\boldsymbol{w}}_{ij}, \qquad \partial_k \boldsymbol{J}^{\boldsymbol{w}} = -(\boldsymbol{I} - \boldsymbol{Q})\,\theta\vartheta\,\boldsymbol{X}^{-1}\,\partial_k(\boldsymbol{T}\boldsymbol{S})\,\boldsymbol{X}^{-1},$$

$$\partial_k(\boldsymbol{T}\boldsymbol{S}) = (\partial_k \boldsymbol{T})\boldsymbol{S} + \boldsymbol{T}(\partial_k \boldsymbol{S})$$

$$(\partial_k \boldsymbol{S})_{ni} \quad = \quad \theta\,\boldsymbol{S}_{ni}\big(\mathbf{1}_{i=k} - \boldsymbol{J}^{\boldsymbol{w}}_{ik} - \boldsymbol{S}_{nk} + (\boldsymbol{S}\boldsymbol{J}^{\boldsymbol{w}})_{nk}\big),$$

$$(\partial_k \boldsymbol{T})_{in} = \boldsymbol{T}_{in}\Big(\theta(\vartheta - 1)\big((\boldsymbol{S}\boldsymbol{J}^{\boldsymbol{w}})_{nk} - \boldsymbol{S}_{nk}\big) - \boldsymbol{J}^{\boldsymbol{w}}_{ik} + \theta\big(\mathbf{1}_{i=k} - \boldsymbol{J}^{\boldsymbol{w}}_{ik} - \boldsymbol{S}_{nk} + (\boldsymbol{S}\boldsymbol{J}^{\boldsymbol{w}})_{nk}\big)\Big).$$

The matrix $\boldsymbol{Q} \equiv \mathbf{1}\boldsymbol{q}'$ captures our choice of world GDP at the approximation point of mean log productivity ($\boldsymbol{\mu_z}$) as the numeraire.

Proof. See Online Appendices G.4 and G.5. □

We again obtain expressions for the first and second moments of the wage distribution in terms of the sufficient statistics of country incomes ($q_i = w_i\bar{\ell}_i$), expenditure shares (S_{ni}), and model parameters. Comparing Proposition 6 for complete markets to Proposition 3 for incomplete markets, the wage Jacobian ($\boldsymbol{J}^{\boldsymbol{w}}$) and Hessian ($\boldsymbol{H}^{\boldsymbol{w}}$) are modified by the presence of state-contingent transfers. First, the wage Jacobian ($\boldsymbol{J}^{\boldsymbol{w}}$) under incomplete markets features an additional $-\boldsymbol{T}$ term in the definition of the matrix $\boldsymbol{X}$ relative to that under complete markets. This additional term captures the direct effect of a productivity shock in a given state ν on expenditure in that state ($\mathcal{W}_n(\nu)$) through wage income ($w_n(\nu)\,\bar{\ell}_n$). Under complete markets, this term is absent, because of the insurance provided by state-contingent transfers ($\mathcal{W}(\nu) = \frac{\mathcal{P}_n(\nu)^{1-1/\gamma}}{\mathbb{E}\left[\mathcal{P}_n(\nu')^{1-1/\gamma}\right]}\mathbb{E}\left[w_n(\nu')\right]\bar{\ell}_n$), which imply that expenditure is no longer equal to wage income in each state $(w_n(\nu)\,\bar{\ell}_n)$.

Second, the wage Jacobian ($\boldsymbol{J}^{\boldsymbol{w}}$) under complete markets includes additional terms in ϑ relative to that under incomplete markets, which again represent the insurance component from state-contingent transfers. From the definitions of ϑ and κ in Proposition 6, this insurance component depends on the coefficient of relative risk aversion (γ), the share of traded goods in consumption expenditure (δ), and the trade elasticity (θ). Third, as under incomplete-markets, wages are determined up to a choice of numeraire, although this numeraire no longer enters the definition of the matrix $\boldsymbol{X}$.

These differences in the wage Jacobian ($\boldsymbol{J}^{\boldsymbol{w}}$) between complete and incomplete markets translate into corresponding differences in the wage Hessian ($\boldsymbol{H}^{\boldsymbol{w}}$). The derivative of the expenditure share matrix ($\partial_k \boldsymbol{S}$) with respect to productivity shocks is the same under both financial market structures. But the derivative of the income share matrix ($\partial_k \boldsymbol{T}$) differs between them. Under incomplete markets, the response of aggregate expenditure to a productivity shock is given by the response of wage income (as captured by the wage Jacobian). In contrast, under complete markets, the response of aggregate expenditure to a productivity shock also includes insurance through the state-contingent transfers.

Despite these differences between complete and incomplete markets, wages vary across states under both financial market structures. All that differs is that countries' marginal utilities of income move jointly under complete markets. We again have analytical expressions for the first and second moments of wages, from which we can recover the first and second moments of all other endogenous variables, and compute the covariances that determine bilateral investments in import and export capacity in Proposition 2. Therefore, we can evaluate the impact of second moment shocks on the first moments of the endogenous variables under complete markets. In our quantitative analysis below, we find a similar set of predictions for the impact of a country's risk profile on trade patterns, income and welfare under both complete and incomplete markets.

7 Multiple Sectors

In our baseline model, we assume a single final goods sector for expositional simplicity. In this section, we show that our approach generalizes to accommodate multiple sectors. In this augmented specification, aggregate uncertainty affects comparative advantage across countries and sectors, as well as aggregate trade shares, income and welfare.

We again assume that the representative agent in country n has constant relative risk aversion preferences ($\mathcal{U}_n$) over the aggregate consumption index ($\mathcal{C}_n$), as in equation (1). We now assume that this aggregate consumption index ($\mathcal{C}_n$) is a Cobb-Douglas function of the sectoral consumption index for each sector k ($\mathbb{C}_n^k$):

$$\mathcal{C}_n = \prod_{k=1}^{K} \left(\mathbb{C}_n^k\right)^{\mu_n^k}, \quad 0 < \mu_n^k < 1, \quad \sum_{k=1}^{K} \mu_n^k = 1, \tag{33}$$

where we again suppress the implicit dependence of variables on state throughout this section to streamline notation.

This sectoral consumption index for importer n ($\mathbb{C}_n^k$) is defined over consumption of the good from each exporter i (c_{ni}^k), according to the CES specification in equation (2). The trading technology within each sector takes the same form as in equation (5). For simplicity, we assume the same elasticity of substitution (ω) and the same expenditure shares for import and export capacity (α, β) across sectors, but it is straightforward to allow these parameters to vary by sector. While we focus on a multi-sector Armington model, our results hold throughout the class of multi-sector trade models with a constant trade elasticity, including Costinot et al. (2012).

The representative agent in country n again has labor endowment $\bar{\ell}_n$. Import and export capacity in each sector are chosen *ex ante* before observing the realizations for countries' productivities in each sector $\{z_n^k\}$. Labor allocated to import or export capacity with a given trade partner in a given sector cannot be reallocated to another trade partner or another sector, moved between import and export capacity, or used for production. After observing the realizations for country productivities in each sector $\{z_n^k\}$, the remaining production labor is allocated *ex post* across trade partners and sectors. Labor market clearing requires that the sum across trade partners and sectors of labor allocated to import capacity, export capacity and production is equal to the economy's labor endowment:

$$\bar{\ell}_i = \sum_{k=1}^{K} \sum_{j=1}^{N} \iota_{ij}^k + \sum_{k=1}^{K} \sum_{n=1}^{N} \left(\ell_{ni}^k + h_{ni}^k \right) \tag{34}$$

The *ex ante* equilibrium in this multi-sector extension exhibits similar properties as in our baseline single-sector model. From cost minimization and zero profits, total employment in import and export capacity across all sectors are constant shares of the economy's labor endowment:

$$\sum_{k=1}^{K} \sum_{j=1}^{N} \iota_{ij}^k = \alpha \bar{\ell}_i, \qquad \sum_{k=1}^{K} \sum_{n=1}^{N} h_{ni}^k = \beta \bar{\ell}_i. \tag{35}$$

where these shares are determined by the Cobb-Douglas shares of import and export capacity (α, β) in sectoral expenditures.

Bilateral employments in import and export capacity within each sector are proportional to the expectations of trade values within that sector weighted by the stochastic discount factor. Therefore, these bilateral employments in import and export capacity again depend not only on expected values but also on variances and covariances across countries and sectors:

$$\begin{aligned} \iota_{ni}^k \propto_{n,k} \quad & \mathbb{E}\left[\left(\left(\iota_{ni}^k\right)^{-\frac{\alpha}{\delta}} \left(h_{ni}^k\right)^{-\frac{\beta}{\delta}} \frac{w_i \tau_{ni}^k}{z_i^k}\right)^{1-\widetilde{\omega}}\right] \\ & \exp\left(\text{Cov}\left(\ln \mathcal{C}_n^{-\gamma}, \ln\left(\frac{w_i \tau_{ni}^k}{z_i^k}\right)^{1-\widetilde{\omega}}\right)\right) \\ & \times \exp\left(\text{Cov}\left(\ln\left(\frac{\mathcal{W}_n}{\mathcal{P}_n}\right), \ln\left(\frac{w_i \tau_{ni}^k}{z_i^k}\right)^{1-\widetilde{\omega}}\right)\right) \\ & \times \exp\left(-\text{Cov}\left(\ln\left(\mathcal{Q}_n^k\right)^{1-\widetilde{\omega}}, \ln\left(\frac{w_i \tau_{ni}^k}{z_i^k}\right)^{1-\widetilde{\omega}}\right)\right), \end{aligned}$$

$$\begin{aligned} h_{ni}^k \propto_{i,k} \quad & \mathbb{E}\left[\left(\frac{\left(\iota_{ni}^k\right)^{-\alpha/\delta} \left(h_{ni}^k\right)^{-\beta/\delta} \tau_{ni}^k}{\mathcal{Q}_n^k}\right)^{1-\widetilde{\omega}} \mu_n^k \mathcal{W}_n\right] \\ & \times \exp\left(Cov\left(\ln \mathcal{C}_i^{-\gamma}, \ln\left(\frac{\tau_{ni}^k}{\mathcal{Q}_n^k}\right)^{1-\widetilde{\omega}} \mu_n^k \mathcal{W}_n\right)\right) \\ & \times \exp\left(-Cov\left(\ln \mathcal{P}_i, \ln\left(\frac{\tau_{ni}^k}{\mathcal{Q}_n^k}\right)^{1-\widetilde{\omega}} \mu_n^k \mathcal{W}_n\right)\right) \\ & \times \exp\left(Cov\left(\ln\left(\frac{w_i}{z_i^k}\right)^{1-\widetilde{\omega}}, \ln\left(\frac{\tau_{ni}^k}{\mathcal{Q}_n^k}\right)^{1-\widetilde{\omega}} \mu_n^k \mathcal{W}_n\right)\right) \end{aligned}$$

where $\propto_{n,k}$ means up to a multiplicative factor that depends only on n and k; $\propto_{i,k}$ means up to a multiplicative factor that depends only on i and k; and the expectation ($\mathbb{E}[\cdot]$) and covariance ($\text{Cov}(\cdot)$) are taken across states (over the distribution of productivities and bilateral trade costs).

These *ex ante* investments in import and export capacity in turn determine *ex post* bilateral trade shares. Without uncertainty, these bilateral trade shares would depend solely on the realized values which equal expected values of variables (first moment). In contrast, with uncertainty, they also depend on variances and covariances across countries and sectors (second moment). Therefore, in this multi-sector environment, uncertainty now shapes the allocation of resources across both sectors and countries.

8 Data and Parameterization

We implement our baseline single-sector model under both incomplete and complete financial markets using data on national income and bilateral trade.

8.1 Data Sources

We obtain international trade data from the IMF's Direction of Trade Statistics (DOTS). We combine these bilateral trade data with national accounts data on GDP, population, and price indexes from the Penn World Tables (PWT) and the Global Macro Database (GMD). We obtain data on gross output from the World Input-Output Database (WIOD). We exclude small countries (those with populations below 0.05% of the world total), and several conflict-affected countries (Afghanistan, Venezuela, Iraq, Syria, Yemen, Zimbabwe, and Liberia), which have particularly volatile output. The resulting panel covers 106 countries over the period 1993–2017.

The DOTS bilateral trade data do not include countries' expenditures on domestically-produced output. We compute domestic expenditure by subtracting exports from total gross output. Gross output data are available from the World Input-Output Database (WIOD) for a subset of countries and years. For observations not covered by WIOD, we impute gross output by predicting the ratio of gross output to GDP based on country characteristics in the WIOD sample. We then apply these predicted ratios to GDP data to estimate gross output for the remaining observations.

8.2 Model Parameterization

8.2.1 Structural Parameters

We calibrate our model's parameters using empirical moments and standard values from existing research. In the macro literature with CRRA utility, the parameter γ is typically set to 2, balancing the intertemporal elasticity of substitution (EIS) and risk aversion, which are inversely related. In contrast, the finance literature, using Epstein-Zin preferences that separate the EIS and risk aversion, often calibrates γ well above 2 in order to match asset pricing moments. Since we abstract from intertemporal savings decisions and focus on risk aversion, we adopt a value of $\gamma = 10$ following the finance literature (e.g., Collin-Dufresne et al. 2016).

We calibrate the elasticity of substitution (ω) and the overall importance of import and export capacity in consumption expenditure ($\alpha + \beta$) using our model's predictions for the long-run trade elasticity (Section 3) and the short-run trade elasticity (Section 4), as discussed further in Online Appendix I. We assume a long-run trade elasticity of $\theta^{LR} = 4$, which is a central value in the international trade literature. We assume a short-run trade elasticity of $\theta^{SR} = 0.8$, which is in line with the estimates in Boehm et al. (2023).

Given these assumed long-run and short-run trade elasticities, we recover an implied elasticity of substitution of $\omega = 9$ (using $\theta^{SR} = \theta^{LR} / \left(\omega - \theta^{LR}\right)$). This implied elasticity of substitution lies within the range estimated using price and expenditure share data in Feenstra (1994) and Broda and Weinstein (2006). Given this value for ω, we recover the implied overall importance of import and export capacity in final consumption ($\alpha + \beta = 1 - \delta$) from the long-run trade elasticity ($\theta^{LR} = \delta\left(\omega - 1\right)$). We obtain an implied value of $\alpha + \beta = 0.5$, which is broadly in line with the shares of non-traded services in final consumption.[6]

We isolate the separate contributions of import capacity (α) and export capacity (β) by assuming an equal factor intensity for the non-traded services of importing and exporting in final consumption ($\alpha = \beta$). While an equal factor intensity is a natural benchmark, our quantitative results are relatively insensitive to the relative values of α and β for given values of the short and long-run trade elasticities (and hence for a given overall value of $\alpha + \beta$).

8.2.2 Model Inversion

We quantify our model treating productivities as uncertain and bilateral trade costs as deterministic. We recover the mean and variance of log productivities ($\boldsymbol{\mu_z}$, $\boldsymbol{\Sigma_z}$) by inverting our model's general equilibrium conditions for each year, as discussed further in Online Appendix J. Given the empirical evidence of relatively little consumption-risk sharing across countries, we undertake this model inversion using our model's general equilibrium conditions under incomplete markets. We recover bilateral trade costs ($\boldsymbol{\tau}$) by inverting these general equilibrium conditions for our benchmark year of 2017. We follow a similar procedure as for the deterministic Armington model, but take into account that *ex post* trade costs depend on import and export capacity, where these bilateral investments depend on expectations of trade values weighted by the marginal utility of income.

In particular, we measure $\mathcal{W}_{nt}$ with nominal GDP and consumption with real GDP ($\mathcal{C}_{nt} = \mathcal{W}_{nt}/\mathcal{P}_{nt}$). We recover wages from nominal GDP and population ($w_{nt} = \mathcal{W}_{nt}/\overline{\ell}_{nt}$). We recover bilateral import and export capacity (ι_{nit}, h_{nit}) using the property that they are proportional to the expectation of bilateral trade weighted by the marginal utility of income. We solve for the marginal utility of income using real GDP, nominal GDP and model parameters. We equate expectations in our model with their sample counterparts in the data.

Given our solutions for import and export capacity (ι_{nit}, h_{nit}), the *ex post* allocations in our model are isomorphic to a deterministic Armington model with adjusted trade costs $\widetilde{\tau}_{nit} = \iota_{nit}^{-\alpha/\delta} h_{nit}^{-\beta/\delta} \tau_{nit}$ and trade elasticity $\widetilde{\omega} - 1$. Using this property, we recover productivity (z_{nt}) from domestic expenditure shares (s_{nnt}): $z_{nt} = \delta \iota_{nnt}^{-\alpha/\delta} h_{nnt}^{-\beta/\delta} \left(w_{nt} / \left(\mathcal{P}_{nt}^{1/\delta} \mathcal{W}_{nt}^{(\delta-1)/\delta} \right) \right) \left(s_{nnt} \right)^{\frac{1}{\widetilde{\omega}-1}}$. To abstract

[6] Burstein et al. (2003) and Berman et al. (2012) report shares of distribution costs in prices for consumer goods from 40-60 percent. We assume that the non-traded services of import and export capacity are supplied using the same resource endowment as traded goods (labor). However, since a constant fraction of labor is allocated to these non-traded services in equilibrium, we could equivalently assume that they are produced using a separate resource endowment.

from secular productivity growth, we measure log productivity shocks as deviations from country-specific quadratic time trends. We solve for deterministic trade costs for our benchmark year of 2017 using bilateral trade shares (s_{ni} for $n \neq i$) and our normalization that $\tau_{nn} = 1$: $\tau_{ni} = \delta^{-1} \iota_{ni}^{\alpha/\delta} h_{ni}^{\beta/\delta} s_{ni}^{-\frac{1}{\tilde{\omega}-1}} / \left(w_i / \left(z_i \mathcal{P}_n^{1/\delta} \mathcal{W}_n^{(\delta-1)/\delta} \right) \right)$.

8.2.3 Country Productivities

We find that the country productivities recovered from our model inversion exhibit an intuitive pattern, as discussed further in Online Appendix K.1. We find that geographically close countries (such as France and Germany) typically have higher bilateral productivity covariances than geographically remote countries (such as South Korea and Kuwait). Productivity covariances between developed countries tend to be positive, with strong positive covariances within Europe and between Europe and North America. Productivity covariances between developed countries and natural resource-abundant countries in the Middle-East tend to be negative. Productivity covariances between North America and African countries tend to be small in absolute magnitude.

Our variance-covariance matrix for country productivities exhibits a dominant global factor that explains much of the variance in de-trended productivities across countries. We implement a principal components decomposition of our variance-covariance matrix, and evaluate the contribution of each component to the overall variance. We find that this global factor accounts for over 43% of the variance in country productivities. The first five factors collectively account for approximately 80% of the variance in country productivities. In our model, labor is the sole factor of production, which implies that measured country productivities also capture capital stocks and labor wedges. Therefore, this finding of a strong global factor is consistent with existing empirical evidence in the international co-movement literature.

9 Quantitative Results

We now use our quantitative model to evaluate the impact of uncertainty on patterns of trade, income and welfare. In Subsection 9.1, we report results for our baseline specification with incomplete markets. In Subsection 9.2, we compare results under complete and incomplete markets, and examine the interaction between real hedging through import and export capacity and financial hedging through state-contingent claims.

9.1 Uncertainty and Incomplete Markets

In our model of general equilibrium uncertainty and endogenous investments in resilience, changes in the second moment of productivity affect the first moments of trade, income and welfare. We now explore these first moment effects under incomplete markets for three different changes in the global structure of uncertainty. In Subsection 9.1.1, we eliminate the cross-country correlation in productivity shocks. In Subsection 9.1.2, we consider an increase in global uncertainty. In Subsection 9.1.3, we examine an increase in uncertainty in an individual country.

9.1.1 Eliminating Cross-Country Productivity Correlations

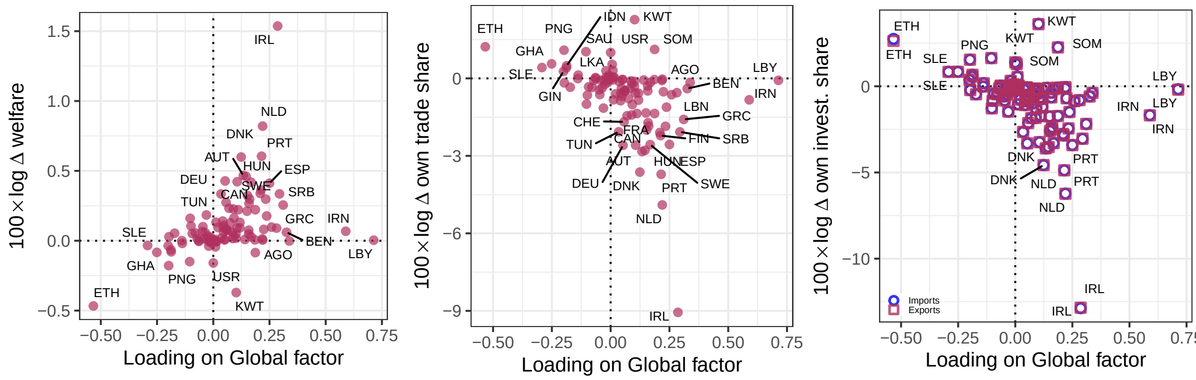
Our first counterfactual examines the implications of eliminating the correlation in the global productivity structure, in order to highlight the importance of cross-country correlations for global trade patterns. We set the off-diagonal terms in the productivity variance-covariance matrix equal to zero, holding constant mean productivity and the variance of productivity. In Figure 1, we show the counterfactual changes in certainty-equivalent welfare (left panel); the change in the domestic expenditure share (middle panel); and the change in domestic investment (right panel) in import capacity (blue circle) and export capacity (red square). We display these counterfactual changes for each country (using ISO 3-digit country codes) against each country's loading on the global productivity factor in the initial equilibrium.

While the counterfactual changes in the endogenous variables depend on general equilibrium linkages between countries in goods markets, and the resulting endogenous adjustments in bilateral investments and relative wages, they are correlated with the loading on the global factor for productivity in the initial equilibrium. We find that welfare tends to increase in countries that load positively on the global factor and decrease in countries that load negatively on the global factor. Intuitively, countries that load positively on the global factor have limited opportunities for hedging through international trade in the initial equilibrium, because their productivity is positively correlated with the global factor. Once we eliminate this positive correlation, these countries experience an expansion in their hedging opportunities through international trade, which leads to a reallocation of investments away from the domestic market (right panel), a decrease in the domestic expenditure share (middle panel), and an increase in welfare (left panel).

In contrast, countries that load negatively on the global factor have substantial opportunities for hedging through international trade in the initial equilibrium, because their productivity is negatively correlated with the global factor. Once we eliminate this negative correlation, these countries experience a reduction in their hedging opportunities through international trade, which leads to a reallocation of investments towards the domestic market (right panel), an increase in the domestic expenditure share (middle panel), and a reduction in welfare (left panel).

Figure 1: Eliminating all Bilateral Productivity Correlations

(a) Welfare (b) Domestic Expenditure Share (c) Domestic Investment



Note: We start from an initial equilibrium with our calibrated variance-covariance matrix of country productivities (Σ_z), bilateral trade costs (τ_{ni}) and labor endowments ($\bar{\ell}_n$) under incomplete markets. We solve for a counterfactual equilibrium in which we set the off-diagonal terms of the variance-covariance matrix equal to zero, holding constant the mean and variance of productivity for each country. The panels of the figure show changes between the counterfactual and initial equilibria in certainty-equivalent welfare (left panel); domestic expenditure share at the approximation point of mean log productivity (middle panel); and domestic investment (right panel) in import capacity (blue circle) and export capacity (red square). Each panel displays the counterfactual changes in these variables against each country's loading on the global productivity factor (first principal component) in the initial equilibrium.

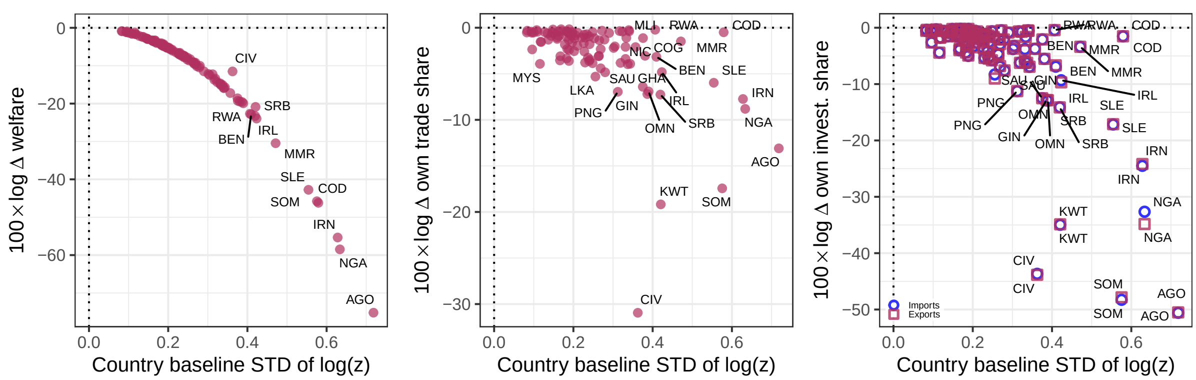
9.1.2 Increasing Global Uncertainty

Our second counterfactual examines the implications of an increase in global uncertainty. We multiply the productivity variance-covariance matrix by 2 and apply a variance correction to keep mean productivity constant. Therefore, this counterfactual isolates the effects of higher global uncertainty, holding constant the mean and correlation structure of productivity. In Figure 2, we show the counterfactual changes in certainty-equivalent welfare (left panel); the change in the domestic expenditure share (middle panel); and the change in domestic investment (right panel) in import capacity (blue circle) and export capacity (red square). We display these counterfactual changes for each country (using ISO 3-digit country codes) against each country's standard deviation of productivity in the initial equilibrium.

While the counterfactual changes in each endogenous variable are again shaped by general equilibrium effects, they exhibit a clear pattern with respect to the initial standard deviation of log productivity. Since the representative agent in each country is risk averse, this increase in global uncertainty reduces certainty-equivalent welfare in all countries. The resulting decline in welfare tends to be larger in countries with more volatile productivity in the initial equilibrium, because a proportional increase in global uncertainty leads to a larger increase in the variance of productivity in these countries (left panel). When sourcing goods in the initial equilibrium, countries trade off sourcing from low-cost suppliers (often from the domestic market where no trade costs are incurred) against diversifying risk by sourcing goods from abroad (where trade costs are incurred but foreign productivity is imperfectly correlated with domestic productivity). As global uncertainty increases (including uncertainty about a country's own productivity), this raises the incentive for risk diversification, which leads to a reduction in domestic investments (right panel) and a reduction in domestic expenditure shares (middle panel).

Figure 2: Mean-Preserving Increase in Variance of Global Productivity Distribution

(a) Welfare (b) Domestic Expenditure Share (c) Domestic Investment



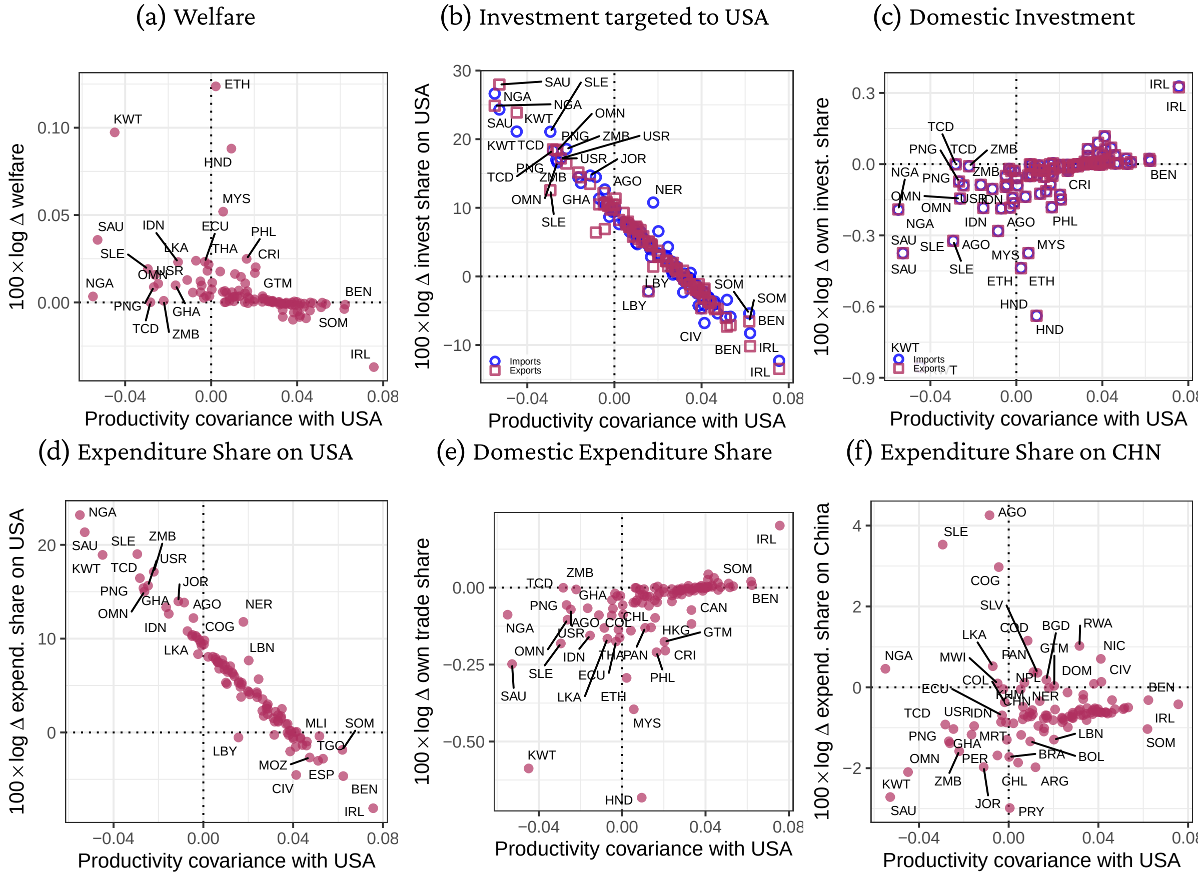
Note: We start from an initial equilibrium with our calibrated variance-covariance matrix of country productivities (Σ_z), bilateral trade costs (τ_{ni}) and labor endowments ($\bar{\ell}_n$) under incomplete markets. We solve for a counterfactual equilibrium in which we double the variances and covariances of productivities and apply a variance correction to keep mean productivity for each country constant. The panels of the figure show changes between the counterfactual and initial equilibria in certainty-equivalent welfare (left panel); domestic expenditure share at the approximation point of mean log productivity (middle panel); and domestic investment (right panel) in import capacity (blue circle) and export capacity (red square). Each panel displays the counterfactual changes in these variables against each country's standard deviation of productivity in the initial equilibrium.

9.1.3 Increasing Country-specific Uncertainty

We next examine the implications of changes in uncertainty for individual countries, motivated by the idea that changes in the institutional environment within countries, such as the protection of property rights, can affect uncertainty over productivity. Our third counterfactual considers an increase in the variance of U.S. productivity, holding constant mean U.S. productivity and the cross-country correlation structure of productivity.[7] Specifically, we double the U.S. variance, and multiply all the covariance terms between each country and the U.S. by the square root of 2 in order to hold the cross-country correlations constant. When productivity takes the form of a factor model as in equation (23) above, this counterfactual can be interpreted as a uniform increase in the U.S. productivity loading across all factors. In Figure 3, we show the results from this counterfactual, where we display the counterfactual change in each variable against each country's productivity covariance with the U.S. in the initial equilibrium.

[7] While we focus on the United States in the paper for brevity, we find a similar pattern of results for changes in uncertainty in other individual countries, such as China.

Figure 3: Mean-Preserving Increase in Variance of U.S. Productivity Distribution



Note: We start from an initial equilibrium with our calibrated variance-covariance matrix of countries' productivities (Σ_z), bilateral trade costs (τ_{ni}) and labor endowments ($\bar{\ell}_n$) under incomplete markets. We solve for a counterfactual equilibrium in which we double the variance of the U.S. productivity distribution, and raise all bilateral covariance terms vis-a-vis the U.S. by the square root of two, to keep bilateral correlations constant. We also apply a correction to the expected value of U.S. log productivity to keep its expected value in levels constant. The panels of the figure show changes between the counterfactual and initial equilibria in certainty-equivalent welfare (top-left panel); investment with the U.S. (top-middle panel) in import capacity (blue circle) and export capacity (red square); domestic investment (top-right panel) in import capacity (blue circle) and export capacity (red square); expenditure share on the U.S. at the approximation point of mean log productivity (bottom-left panel); domestic expenditure share at the approximation point of mean log productivity (bottom-middle panel); and expenditure share on China at the approximation point of mean log productivity (bottom-right panel). Each panel displays the counterfactual changes in these variables against each country's productivity covariance with the U.S. in the initial equilibrium.

While the counterfactual changes in the endogenous variables are again mediated by general equilibrium effects, we find a clear relationship with countries' initial productivity covariance with the U.S. We find that countries that initially have a positive covariance with the U.S. (such as Ireland, Mexico, and Canada) tend to experience a decline in certainty-equivalent welfare, because this increase in the dispersion of U.S. productivity makes trade with the U.S. less attractive for risk diversification (top-left panel). Consequently, these positive-covariance countries typically reduce investment towards the U.S. (top-middle panel) and increase domestic investment (top-right panel). This reallocation of investment tends to reduce the expenditure share on the U.S. (bottom-left panel) and increase the domestic expenditure share (bottom-middle panel) for these positive-covariance countries. Finally, while most of the adjustment occurs through reallocations towards the domestic market, some positive-covariance countries raise their expenditure share on third countries such as China (bottom-right panel).

In contrast, countries that initially have a negative covariance with the U.S. (such as Kuwait and Saudi Arabia) tend to experience a rise in certainty-equivalent welfare, because this increase in the dispersion of U.S. productivity makes trade with the U.S. more attractive for risk diversification (top-left panel). Consequently, these negative-covariance countries typically raise investment towards the U.S. (top-middle panel) and decrease domestic investment (top-right panel). This reallocation of investment tends to increase the expenditure share on the U.S. (bottom-left panel) and decrease the domestic expenditure share (bottom-middle panel) for these negative-covariance countries. Finally, while most of the adjustment again occurs through reallocations with the domestic market, some negative-covariance countries reduce their expenditure share on third countries such as China (bottom-right panel).

These changes in real hedging opportunities induced by the increase in the variance of U.S. productivity, and the resulting reallocations of investment across markets, have general equilibrium effects for the terms of trade between countries. We find that the lower demand for U.S. goods from positive-covariance countries dominates the higher demand for U.S. goods from negative-covariance countries, because the countries with a positive covariance with the U.S. in the initial equilibrium account for a larger share of world GDP than those with a negative covariance with the U.S. As a result, the relative price of U.S. goods falls in the new equilibrium, which tends to increase the expenditure share on the U.S. for all countries, such that the unweighted average change in the expenditure share on the U.S. is above zero in the bottom-left panel.

9.2 Complete Versus Incomplete Markets

We now compare our quantitative results under complete and incomplete markets. We first repeat our three counterfactuals for changes in global and country-specific uncertainty in the previous subsection under complete markets. We next show that the welfare gains from allowing for real hedging through endogenous bilateral investments in import and export capacity are of a comparable magnitude to those from financial hedging through state-contingent claims. Finally, we find that real and financial hedging are complementary, such that welfare gains from real hedging are larger under complete markets than under incomplete markets.

9.2.1 Counterfactuals for Changes in Global and Country-Specific Uncertainty

In Online Appendix K.2.2, we report the results of our three counterfactuals under complete markets: (i) Eliminating cross-country productivity correlations; (ii) Increasing global uncertainty; (iii)

Increasing country-specific uncertainty. Consistent with our similar analytical results above, we find a similar pattern of quantitative results under complete and incomplete markets, with a few modifications as discussed below.

In our first counterfactual for eliminating the correlation in the global productivity structure, we find that the absolute magnitude of the changes in welfare is larger under complete markets than under incomplete markets. This pattern of results reflects a complementarity between the welfare gains from trade in goods and financial claims. Intuitively, the gains from trading financial claims are higher when the costs of trading goods are lower, because goods trade is used to finance the net trade in financial claims (and vice versa). This complementarity implies that changes in real hedging opportunities from eliminating the correlation in the global productivity structure have larger effects on welfare under complete markets than under incomplete markets.

In our second counterfactual for a global increase in uncertainty, we find that the magnitudes of the changes in welfare are smaller under complete markets than under incomplete markets, because state-contingent claims provide an alternative source of insurance under complete markets, which is not available under incomplete markets. In our third counterfactual for an increase in uncertainty in the United States, we find a different pattern of correlation between welfare changes and the initial productivity covariance with the United States under complete and incomplete markets. The reason is that the initial productivity covariance with the United States is no longer fully informative about consumption exposure to risk in the United States under complete markets, because of insurance through state-contingent claims in the initial equilibrium. Nevertheless, the incentives for real hedging are similar under both assumptions about trade in financial assets. Therefore, we find a similar pattern of correlation between changes in bilateral investments and the initial productivity covariance with the United States under both complete and incomplete markets.

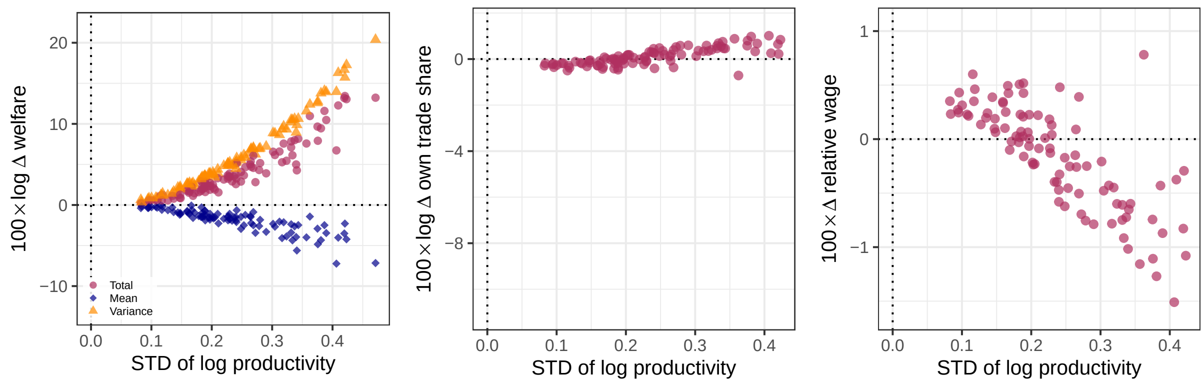
9.2.2 Welfare Gains from Real and Financial Hedging

We now compare the gains in certainty-equivalent welfare from financial hedging and real hedging. In Row (a) of Figure 4, we evaluate the impact of financial hedging. Starting from the complete markets equilibrium, we undertake a counterfactual in which we shut down trade in financial assets, but keep investments in import and export capacities fixed at their baseline levels in the complete markets equilibrium. In Row (b) of Figure 4, we evaluate the impact of real hedging. Starting from the complete markets equilibrium, we undertake a counterfactual in which we exogenously set bilateral investments equal to their values in the deterministic equilibrium, but continue to allow for trade in financial claims. In both rows, we report values in the complete markets equilibrium minus values in the counterfactual equilibrium, such that the top row shows the gains from financial hedging, while the bottom row shows the gains from real hedging. The left column shows the change in overall certainty-equivalent welfare and in its mean and variance components, calculated using equation

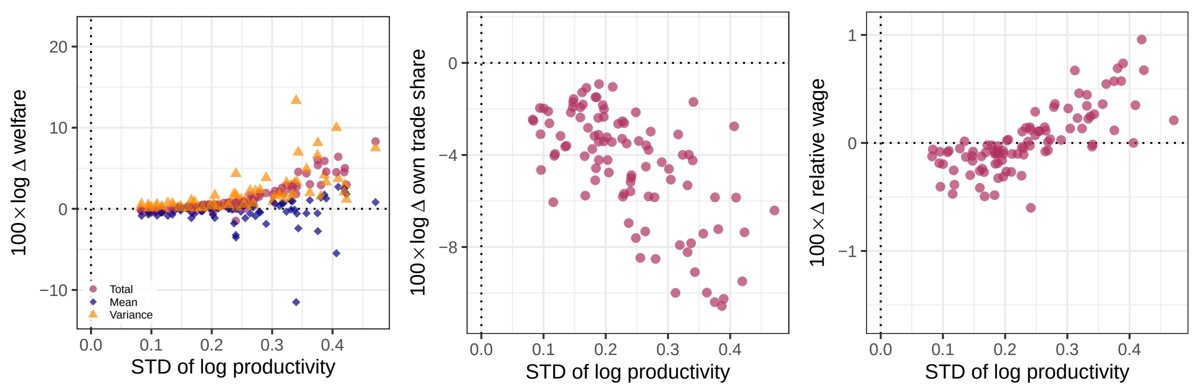
(22); the middle column displays the change in domestic expenditure shares; and the right column shows the change in wages.

Figure 4: Welfare Gains from Financial and Real Hedging

(a) Allowing Trade in Assets



(b) Allowing Endogenous Investments



Note: Row (a) shows a counterfactual in which we start from the complete markets equilibrium and shut down trade in financial assets, but keep investments in import and export capacities fixed at their baseline levels in the complete markets equilibrium. Row (b) shows a counterfactual in which we start from the complete markets equilibrium and exogenously set bilateral investments equal to their values in the deterministic equilibrium, but continue to allow for trade in financial claims. In each panel, we report values in the complete markets equilibrium minus values in the counterfactual equilibrium. Left column shows the changes in overall certainty-equivalent welfare from equation (22) (maroon circle), the changes in its first moment component (blue diamond), and the changes in its second moment component (orange triangle). Middle column shows the changes in domestic expenditure shares, evaluated at the approximation point of countries' mean log productivities. Right column shows the changes in relative wages, defined as the log change in wages net of the expenditure-share-weighted average of its supplier's wage changes: $\Delta \ln w_n - \sum_{k=1}^{N} S_{nk} \Delta \ln w_k$, where the expenditure-shares (S_{nk}) are held constant at their values in the complete markets equilibrium, evaluated at the approximation point of countries' mean log productivities.

Although the welfare gains from real hedging are smaller than those from financial hedging, we find that both can be substantial, particularly for countries with high standard deviations of productivity in the initial equilibrium (left column comparing the two rows). When engaging in financial hedging, countries trade-off lower expected welfare versus insurance against bad states of the world.

Therefore, we find that the increase in certainty-equivalent welfare for all countries is accompanied by a negative mean component that is more than offset by a positive variance component.

In contrast, real hedging adjusts bilateral investments in import and export capacity (and hence bilateral trade costs) with each trade partner to take account of uncertainty. These adjustments in bilateral investments again involve a trade-off between expected welfare and insurance against bad states of the world. But they also involve endogenous changes in trade costs, which have general equilibrium effects on country wages (the terms of trade). Therefore, although there is negative mean component and a positive variance component from real hedging for most countries, this is not always the case, and some countries have positive mean and variance components.

Financial hedging equates the ratio of countries' marginal utilities of income across states of the world through state-contingent transfers. Since these transfers affect expenditure across all bilateral trade partners, we find relatively modest effects on domestic expenditure shares, which rise for some countries and fall for others (middle column, top row). In contrast, when countries engage in real hedging, they increase bilateral investments in import and export capacity with trade partners that have worse expected characteristics, in order to be able to trade with these partners when they experience favorable realizations of characteristics. Increased bilateral investments with those trade partners come at the expense of reduced bilateral investments with trade partners with favorable expected characteristics. Since domestic trade is not subject to trade costs, it typically has favorable expected characteristics. Therefore, real hedging systematically involves a reduction in domestic investments and a lower domestic expenditure share (middle column, bottom row).

When engaging in financial hedging, more uncertain countries pay transfers to less uncertain countries, in order to obtain insurance against bad states of the world. These larger transfers from countries with greater productivity uncertainty reduce expected expenditure in those countries, which in general equilibrium leads to a reduction in wages in those countries. Therefore, we find a negative relationship between changes in wages and productivity uncertainty when moving from incomplete to complete financial markets (right column, top row). In contrast, when engaging in real hedging, countries with greater productivity uncertainty invest more in reducing bilateral trade costs with trade partners whose marginal costs or market demand are negatively correlated with their own marginal costs or market demand. These reductions in bilateral trade costs raise the demand for the goods produced by countries with greater productivity uncertainty. Therefore, we find a positive relationship between changes in wages and productivity uncertainty when we allow bilateral investments to adjust to uncertainty (right column, bottom row).

Therefore, we find that the welfare gains from real hedging can be sizable relative to those from financial hedging. Trade, income and welfare are shaped by uncertainty under both complete and incomplete markets. But financial and real hedging operate through different mechanisms, and hence affect trade, income and welfare in different ways.

9.2.3 Complementarity Between Real and Financial Hedging

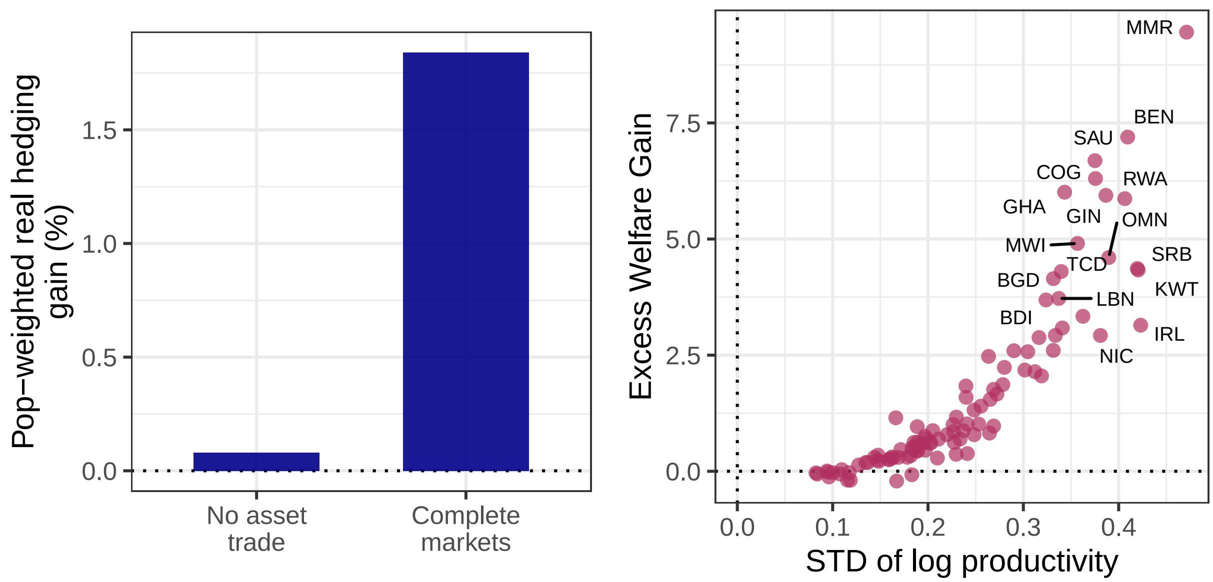
We next examine the extent to which real and financial hedging are substitutes or complements for one another. We compute the welfare gains from real hedging, measured as the change in certainty-equivalent welfare between an equilibrium under uncertainty with endogenous bilateral investments and an equilibrium under uncertainty with exogenous bilateral investments held constant at their deterministic values. We compute this welfare gain from real hedging under both incomplete and complete financial markets.

In Panel (a) of Figure 5, we show the population-weighted average of countries' welfare gains from real hedging in both cases. Incomplete financial markets and exogenous bilateral investments are both sources of inefficiency. Therefore, in general, the welfare gain from removing one of these frictions can be either larger or smaller once the second friction is removed, by the usual logic of the economics of second-best. In practice, we find that the population-weighted average welfare gains from real hedging are larger under complete markets than under incomplete markets. This finding of a complementarity between real and financial hedging reflects the fact that the gains from reducing bilateral trade costs through investments in import and export capacity are larger with asset trade than without it, because these bilateral trade cost reductions make it less costly to exchange the goods used to finance the asset trade.

In Panel (b) of Figure 5, we plot the welfare gain from real hedging under complete markets minus that under incomplete markets for each country against the standard deviation of country log productivity from our model inversion. Consistent with the logic of the economics of second-best, the presence of more than one friction implies that this difference can be negative for some countries, such that the welfare gains from real hedging are larger for these countries under incomplete markets. However, for almost all countries, we find that this difference is positive, such that the welfare gains from real hedging are larger under complete markets. Furthermore, the extra welfare gains from real hedging under complete markets are larger for more uncertain countries. Taken together, these findings provide evidence of a complementarity between real and financial hedging, which is greater for countries with more uncertain production environments.

Figure 5: Complementarity Between Real and Financial Hedging

(a) Population-Weighted Real Hedging Gains (b) Excess Welfare Gains by Country



Note: Panel (a) shows that the population-weighted average welfare gain from real hedging, defined as the change in certainty-equivalent welfare when moving from fixed investments in import and export capacity held constant at their deterministic values to equilibrium investments in import and export capacity under uncertainty. We show this population-weighted average welfare gain from real hedging under both incomplete markets (left column) and complete markets (right column). Panel (b) shows the complete markets welfare gains from real hedging minus the incomplete markets welfare gains from real hedging for each country individually against the standard deviation of country log productivity.

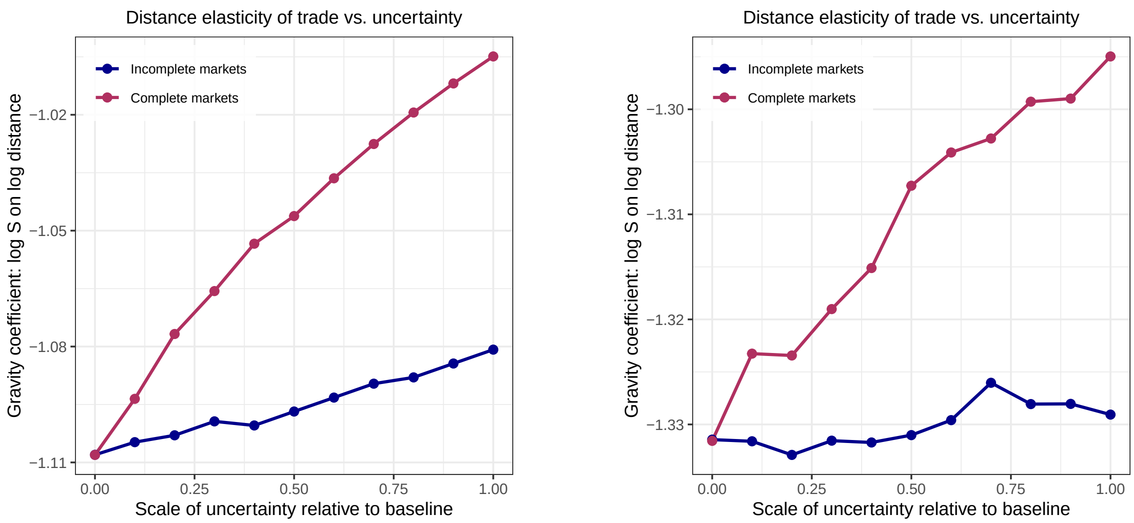
9.2.4 Gravity in an Uncertain World

Finally, we show that uncertainty and endogenous investments in resilience shape the interpretation of the gravity equation in international trade, because the bilateral co-movement in countries' productivities is correlated with the bilateral distance between countries.

We start by solving for bilateral expenditure shares (S_{ni}) at countries' mean log productivities ($\boldsymbol{\mu}_z$) in our benchmark year of 2017, which corresponds to the point around which we take our second-order Taylor series expansion in the system of general equilibrium conditions in Propositions 3 and 6. We compute these bilateral expenditure shares for both our baseline variance-covariance matrix for country productivities ($\boldsymbol{\Sigma}_z$) and for alternative variance-covariance matrices that are constant multiples of our baseline matrix. These bilateral expenditure shares evaluated at mean countries' mean log productivities ($\boldsymbol{\mu}_z$) take different values under complete and incomplete markets, because the system of general equilibrium conditions differs across these two alternative assumptions about asset trade. These bilateral expenditure shares also take different values for alternative variance-covariance matrices for country productivities, because the structure of global uncertainty affects both real and financial hedging, and hence bilateral expenditure shares.

Figure 6: Gravity in an Uncertain World

(a) Top-30 Largest Countries (b) All 106 Countries



Note: Figure shows the estimated coefficient from a regression of log bilateral expenditure shares (S_{ni}) on log bilateral distance for our benchmark year of 2017, controlling for exporter fixed effects and importer fixed effects and dropping bilateral observations with trade shares < 0.001%; Panel (a) shows results for the 30 largest countries in terms of 2017 GDP; Panel (b) shows results for all 106 countries; red line shows the estimated distance coefficient under complete financial markets; blue line shows the estimated distance coefficient under incomplete financial markets; estimated coefficients show results using our baseline variance-covariance matrix (at the value of one on the horizontal axis) and results after multiplying our baseline variance-covariance by a fraction less than one (a value less than one on the horizontal axis). We recover our baseline variance-covariance matrix from an inversion of the model with endogenous investments and incomplete financial markets, as discussed in Online Appendix J. When productivities are deterministic (when uncertainty relative to the baseline on the horizontal axis is equal to zero), the incomplete and complete markets models align.

We regress the log of these bilateral expenditures on log bilateral geographical distance between countries, including importer and exporter fixed effects. Figure 6 displays the estimated distance coefficient for incomplete and complete markets and for different levels of productivity uncertainty. Panel (a) shows results for the 30 largest countries in GDP in 2017, which together account for 89 percent of world GDP and 64 percent of world trade. Panel (b) shows results for all 106 countries. A number of features are apparent. First, we find that allowing for trade in financial assets affects the distance elasticity, because insurance through state-contingent claims affects expenditure, wages and bilateral investments in import and export capacity, and hence changes bilateral trade shares. Second, we find that this impact of allowing asset trade on the distance coefficient varies with the country composition of the sample, because sample composition affects the structure of the variance-covariance matrix, and hence the scope for real and financial hedging.

Third, for a given assumption about asset trade and country composition of the sample, we find that increases in productivity uncertainty tend to reduce the absolute magnitude of the distance coefficient, because productivities are typically more positively correlated between nearby countries. Greater productivity uncertainty raises the incentive for real hedging away from these nearby trade partners towards more remote trade partners with lower productivity correlations. The resulting change in bilateral investments in import and export capacity tends to reduce bilateral trade costs at long distances relative to short distances, which in turn increases trade at long distances relative to short distances, thereby reducing the absolute magnitude of the estimated distance coefficient. Therefore, uncertainty and endogenous investments in resilience shape the interpretation of the gravity equation in international trade.

10 Conclusions

Recent years have seen growing concerns about uncertainty and resilience. We develop a tractable approach to incorporating general equilibrium uncertainty into the class of quantitative trade models with a constant trade elasticity. We provide an analytical characterization of the first and second moments of the distributions of the endogenous variables that holds throughout this class of models, allowing for many asymmetric countries, an arbitrary network of bilateral trade costs, and a general distribution of aggregate shocks. Our characterization holds up to a second-order approximation without imposing a specific functional form for the distribution of exogenous shocks.

We allow agents to make endogenous *ex ante* investments in import and export capacity that determine resilience in the face of this general equilibrium uncertainty. In the special case of no uncertainty, our model is isomorphic to a deterministic constant elasticity quantitative trade model. More generally, in the presence of uncertainty, bilateral investments in import and export capacity solve a portfolio problem, in which bilateral trade shares depend not only on the expected values of the endogenous variables but also on their variances and covariances. The general equilibrium of our model is a fixed point at which bilateral investments in import and export capacity based on the first and second moments of the endogenous variables induce distributions of these endogenous variables that are consistent with these first and second moments.

Our modelling of endogenous investments in resilience under the shadow of general equilibrium uncertainty highlights that changes in the second moments of exogenous primitives can have first moment implications for trade patterns, income and welfare. The opening of the closed economy to trade affects welfare through both the mean and volatility of real income. A modified version of the ACR formula holds for the first moment component of welfare gains, modified to take account of endogenous changes in domestic investments between the closed and open economies. The volatility of real income can either rise or fall following the opening of trade, depending on the covariance structure of domestic and foreign shocks. Nevertheless, regardless of whether real income volatility rises or falls, the opening of trade is revealed-preferred to autarky.

We implement our approach under both financial autarky (incomplete markets) and trade in financial assets (complete markets). In both cases, countries' risk profiles become determinants of trade patterns, income and welfare. Complete markets ensure that the ratios of countries' marginal utilities of income are equalized across states of the world for any network of bilateral investments in import and export capacity. However, agents are not indifferent over alternative networks of bilateral investments, and choose these bilateral investments based on the cross-country correlation of costs and revenues. We show that changes in the global structure of uncertainty have heterogeneous effects across countries, depending on the extent to which they raise or reduce countries' ability to engage in real hedging through bilateral investments in import and export capacity.

Countries trade off sourcing domestically (where no trade costs are incurred) versus diversifying risk by sourcing goods from abroad (where trade costs are incurred but foreign productivity is imperfectly correlated with domestic productivity). Eliminating the correlation structure in country productivities raises welfare in countries that load positively on the global factor for productivity, because eliminating this positive correlation raises the ability of these countries to engage in real hedging through trade (with the converse holding for countries that load negatively on the global factor). An increase in global uncertainty reduces welfare and increases the incentive for risk diversification, which reduces both domestic investments and domestic expenditure shares.

We show that the welfare gains from real hedging through bilateral investments are sizable relative to those from financial hedging through state-contingent claims. We find that real and financial hedging are complementary, with larger welfare gains from real hedging under complete markets than under incomplete markets. Overall, we provide a tractable methodology that can be used to evaluate the quantitative implications of a range of counterfactual questions in international trade concerned with changes in the global structure of uncertainty.

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